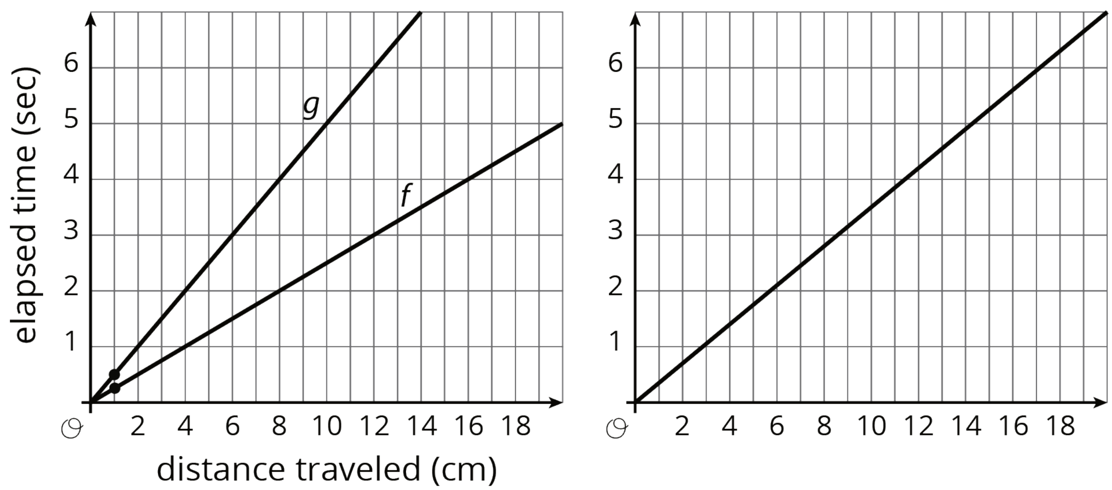
NAME DATE PERIOD

Unit 3, Lesson 1: Understanding Proportional Relationships

Let’s study some graphs.

1.1: Notice and Wonder: Two Graphs

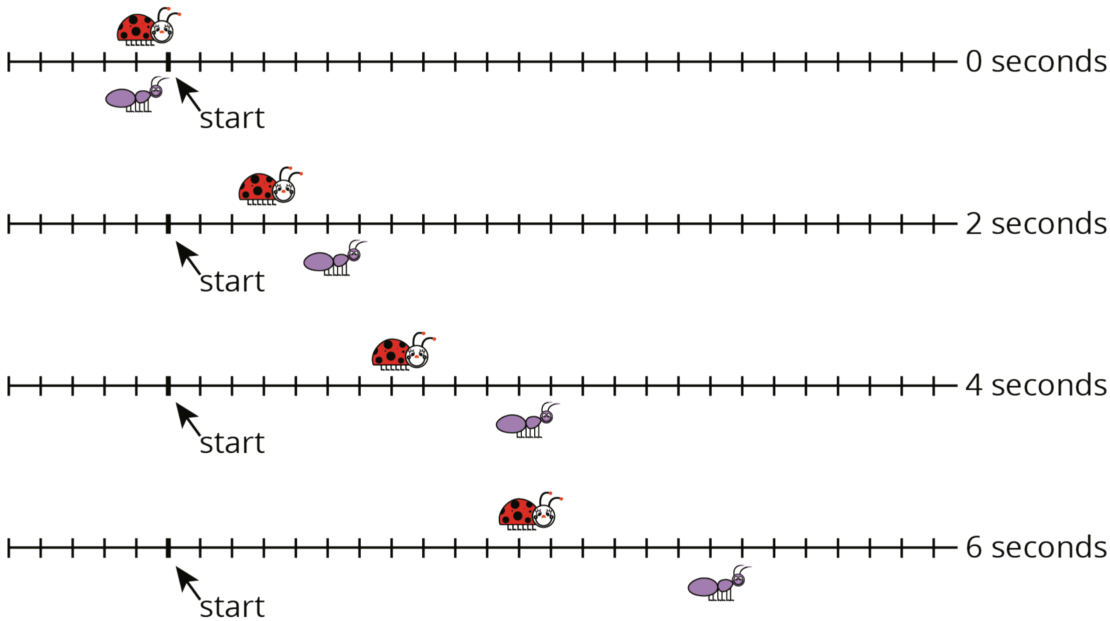


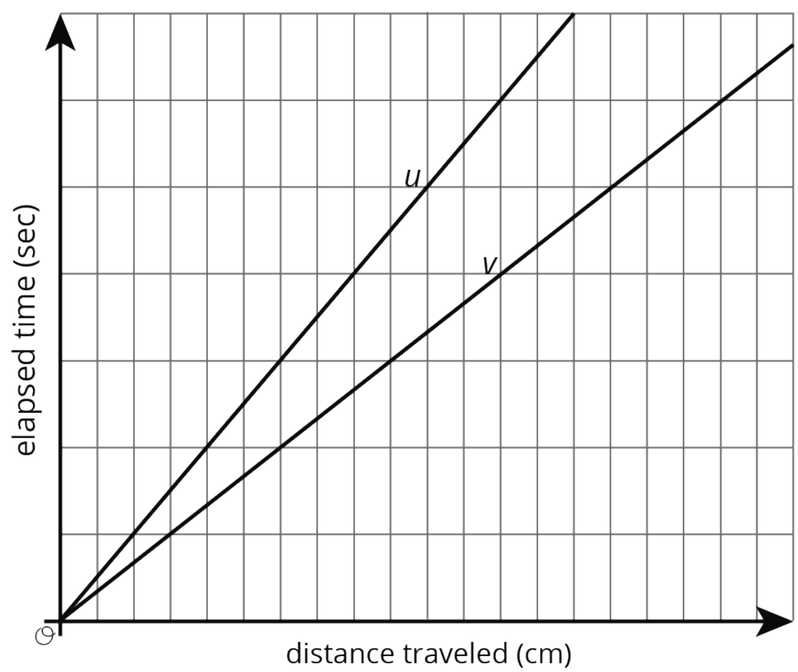
What do you notice? What do you wonder?

NAME DATE PERIOD

1.2: Moving Through Representations

A ladybug and ant move at constant speeds. The diagrams with tick marks show their positions at different times. Each tick mark represents 1 centimeter.





NAME DATE PERIOD

1. Lines u and v also show the positions of the two bugs. Which line shows the ladybug's movement? Which line shows the ant's movement? Explain your reasoning.

2. How long does it take the ladybug to travel 12 cm? The ant?

3. Scale the vertical and horizontal axes by labeling each grid line with a number. You will need to use the time and distance information shown in the tick-mark diagrams.

4. Mark and label the point on line u and the point on line v that represent the time and position of each bug after travelling 1 cm.

Are you ready for more?

1. How fast is each bug traveling?

2. Will there ever be a time when the purple bug (ant) is twice as far away from the start as the red bug (ladybug)? Explain or show your reasoning.

1.3: Moving Twice as Fast

Refer to the tick-mark diagrams and graph in the earlier activity when needed.

1. Imagine a bug that is moving twice as fast as the ladybug. On each tick-mark diagram, mark the position of this bug.

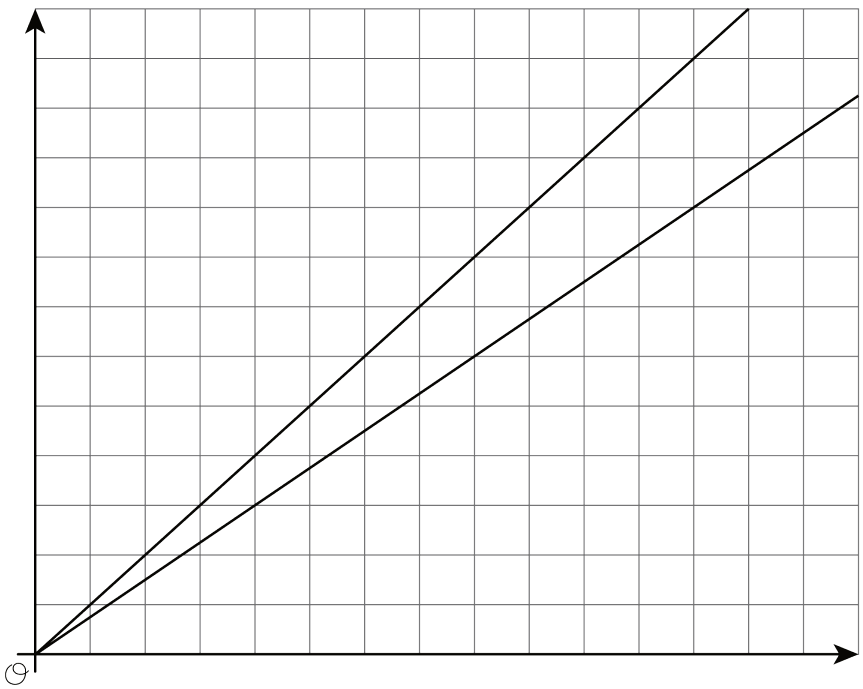
2. Plot this bug's positions on the coordinate axes with lines u and v, and connect them with a line.

3. Write an equation for each of the three lines.

NAME DATE PERIOD

Lesson 1 Summary

Graphing is a way to help us make sense of relationships. But the graph of a line on a coordinate axes without scale or labels isn't very helpful. For example, let's say we know that on longer bike rides Kiran can ride 4 miles every 16 minutes and Mai can ride 4 miles every 12 minutes. Here are the graphs of these relationships:

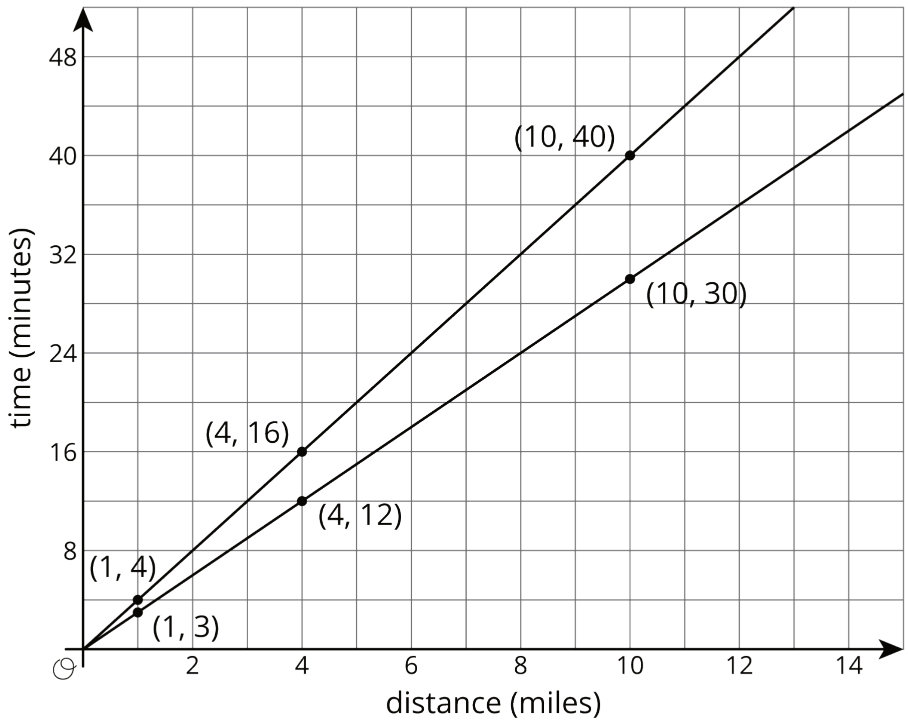


Without labels we can't even tell which line is Kiran and which is Mai! Without labels and a scale on the axes, we can't use these graphs to answer questions like:

1. Which graph goes with which rider?
2. Who rides faster?
3. If Kiran and Mai start a bike trip at the same time, how far are they after 24 minutes?
4. How long will it take each of them to reach the end of the 12 mile bike path?

NAME DATE PERIOD

Here are the same graphs, but now with labels and scale:



Revisiting the questions from earlier:

1. Which graph goes with each rider? If Kiran rides 4 miles in 16 minutes, then the point $(4, 16)$ is on his graph. If he rides for 1 mile, it will take 4 minutes. 10 miles will take 40 minutes. So the upper graph represents Kiran's ride. Mai's points for the same distances are $(1, 3)$, $(4, 12)$, and $(10, 30)$, so hers is the lower graph. (A letter next to each line would help us remember which is which!)

2. Who rides faster? Mai rides faster because she can ride the same distance as Kiran in a shorter time.

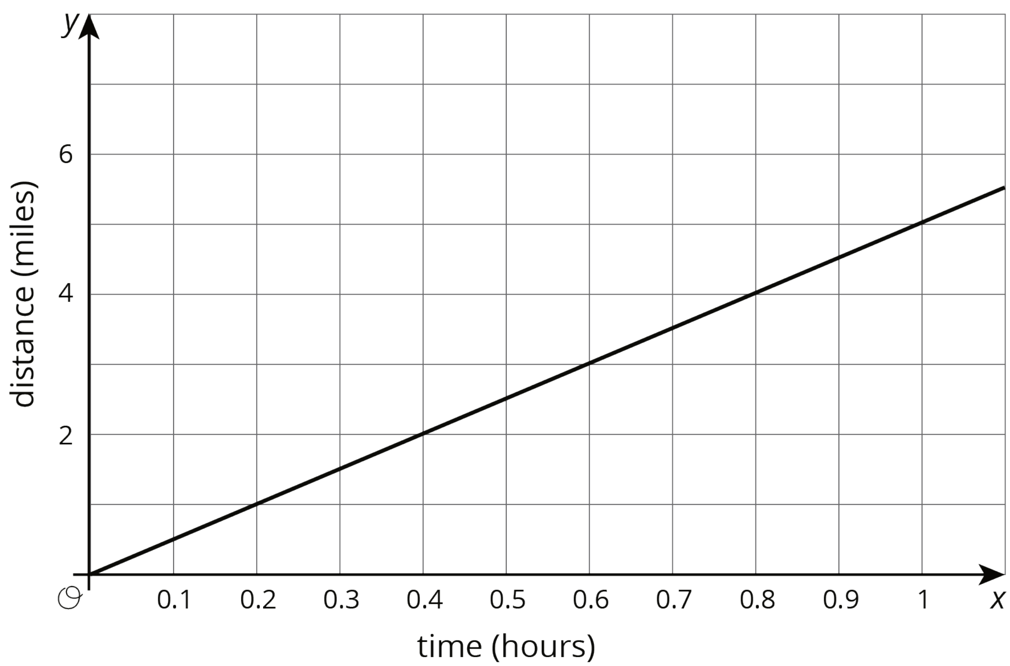
3. If Kiran and Mai start a bike trip at the same time, how far are they after 20 minutes? The points on the graphs at height 20 are 5 miles for Kiran and a little less than 7 miles for Mai.

4. How long will it take each of them to reach the end of the 12 mile bike path? The points on the graphs at a horizontal distance of 12 are 36 minutes for Mai and 48 minutes for Kiran. (Kiran's time after 12 miles is almost off the grid!)

NAME DATE PERIOD

Unit 3, Lesson 1: Understanding Proportional Relationships

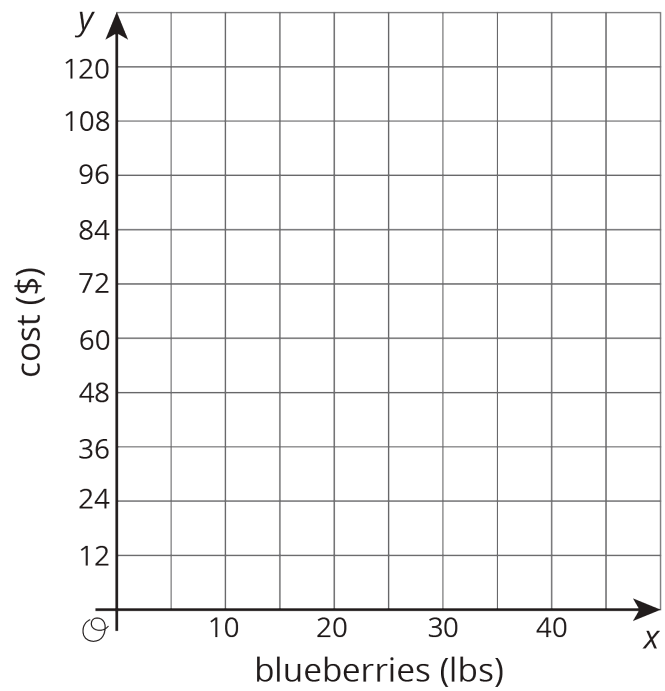
1. Priya jogs at a constant speed. The relationship between her distance and time is shown on the graph. Diego bikes at a constant speed twice as fast as Priya. Sketch a graph showing the relationship between Diego's distance and time.



2. A you-pick blueberry farm offers 6 lbs of blueberries for $16.50.

NAME DATE PERIOD

Sketch a graph of the relationship between cost and pounds of blueberries.



3. A line contains the points $(-4, 1)$ and $(4, 6)$. Decide whether or not each of these points is also on the line:

   a. $(0, 3.5)$

   b. $(12, 11)$

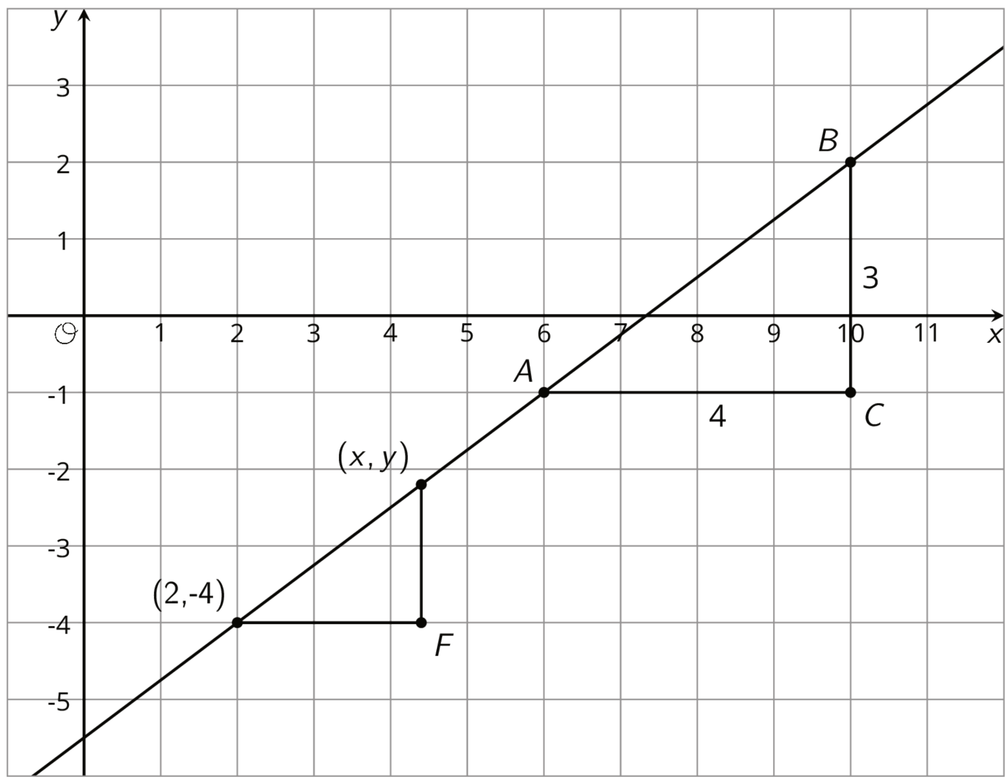
   c. $(80, 50)$

   d. $(-1, 2.875)$

   (from Unit 2, Lesson 12)

4. The points $(2, -4)$, (x, y), A, and B all lie on the line. Find an equation relating x and y.

NAME DATE PERIOD



(from Unit 2, Lesson 11)

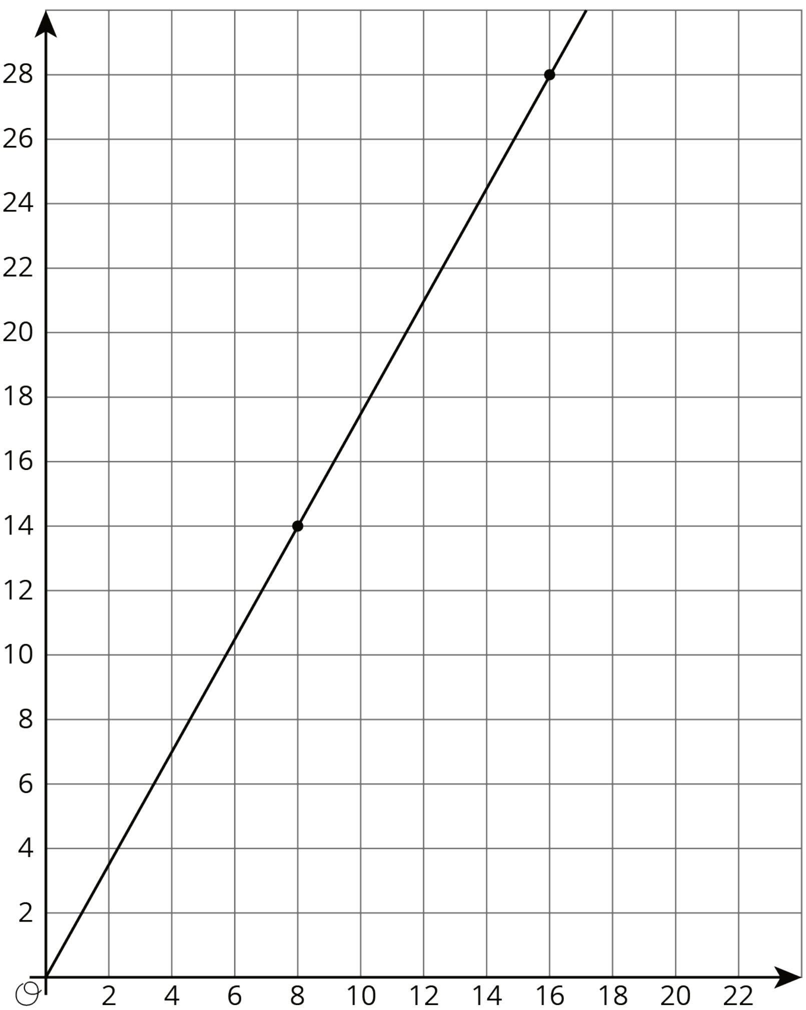
NAME DATE PERIOD

Unit 3, Lesson 2: Graphs of Proportional Relationships

Let's think about scale.

2.1: An Unknown Situation

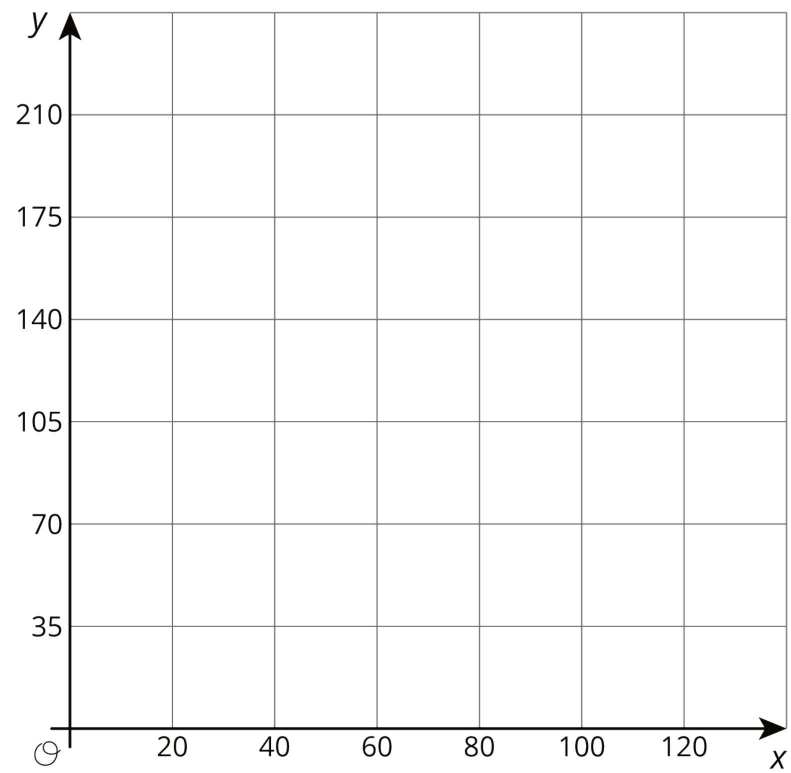
Here is a graph that could represent a variety of different situations.



NAME DATE PERIOD

1. Write an equation for the graph.

2. Sketch a new graph of this relationship.



2.2: Card Sort: Proportional Relationships

You teacher will give you 12 graphs of proportional relationships.

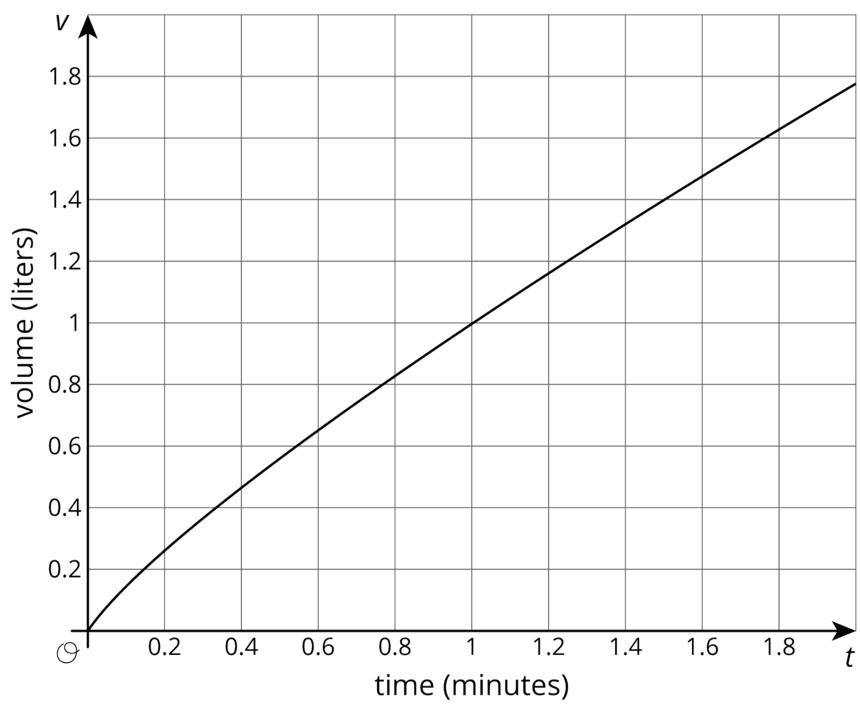
1. Sort the graphs into groups based on what proportional relationship they represent.

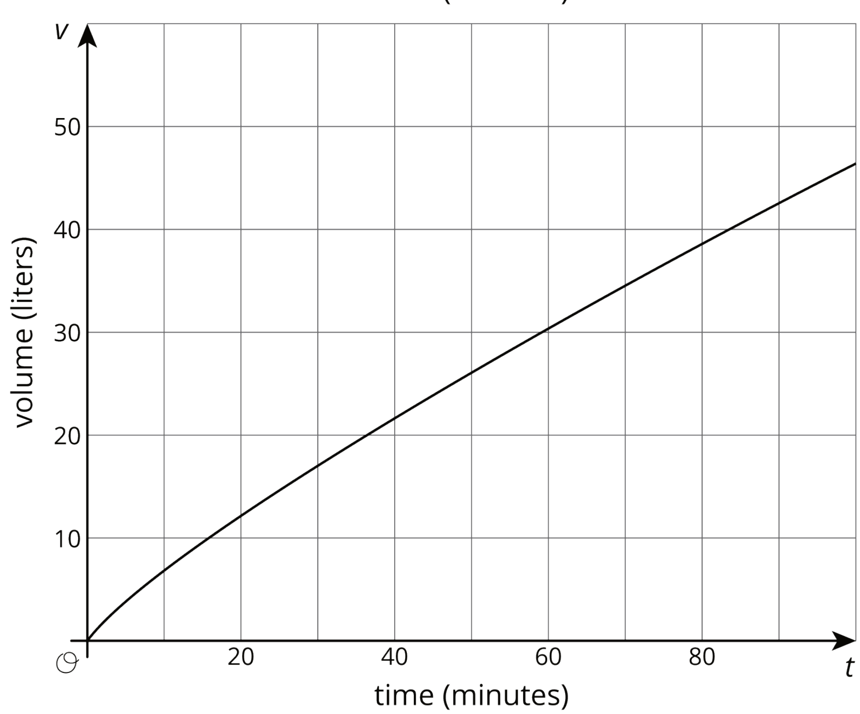
2. Write an equation for each *different* proportional relationship you find.

NAME DATE PERIOD

2.3: Different Scales

Two large water tanks are filling with water. Tank A is not filled at a constant rate, and the relationship between its volume of water and time is graphed on each set of axes. Tank B is filled at a constant rate of $\frac{1}{2}$ liters per minute. The relationship between its volume of water and time can be described by the equation $v = \frac{1}{2}t$, where t is the time in minutes and v is the total volume in liters of water in the tank.





NAME DATE PERIOD

1. Sketch and label a graph of the relationship between the volume of water v and time t for Tank B on each of the axes.

2. Answer the following questions and say which graph you used to find your answer.
    a. After 30 seconds, which tank has the most water?

    b. At approximately what times do both tanks have the same amount of water?

    c. At approximately what times do both tanks contain 1 liter of water? 20 liters?

Are you ready for more?

A giant tortoise travels at 0.17 miles per hour and an arctic hare travels at 37 miles per hour.

1. Draw separate graphs that show the relationship between time elapsed, in hours, and distance traveled, in miles, for both the tortoise and the hare.

2. Would it be helpful to try to put both graphs on the same pair of axes? Why or why not?

3. The tortoise and the hare start out together and after half an hour the hare stops to take a rest. How long does it take the tortoise to catch up?

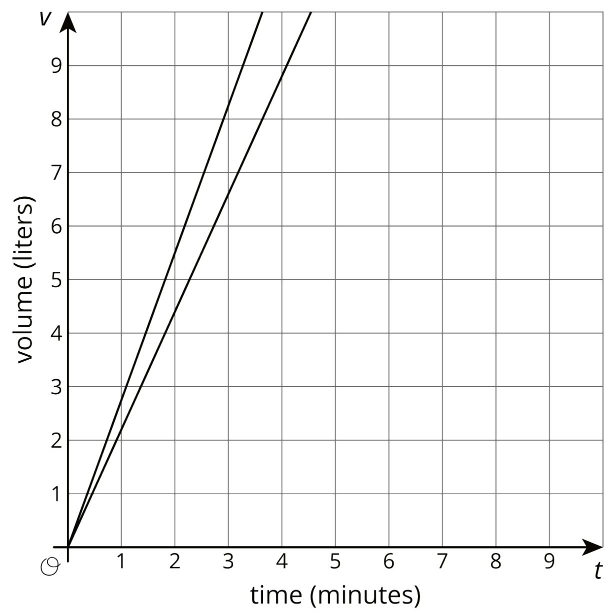
Lesson 2 Summary

The scales we choose when graphing a relationship often depend on what information we want to know. For example, say two water tanks are filled at different constant rates. The relationship between time in minutes t and volume in liters v of tank A is given by $v = 2.2t$. For tank B the relationship is $v = 2.75t$.

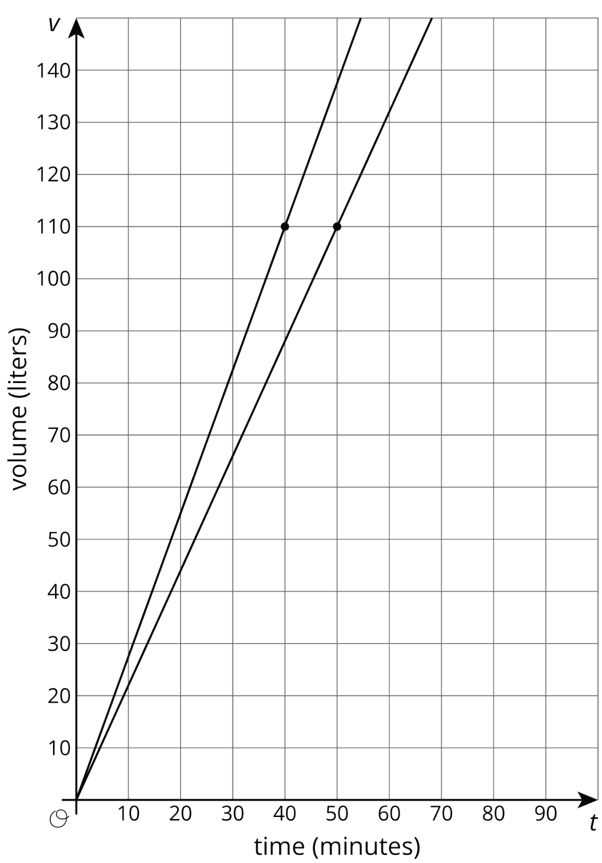
NAME DATE PERIOD

These equations tell us that tank A is being filled at a constant rate of 2.2 liters per minute and tank B is being filled at a constant rate of 2.75 liters per minute.

If we want to use graphs to see at what times the two tanks will have 110 liters of water, then using an axis scale from 0 to 10, as shown here, isn't very helpful.



If we use a vertical scale that goes to 150 liters, a bit beyond the 110 we are looking for, and a horizontal scale that goes to 100 minutes, we get a much more useful set of axes for answering our question.



Now we can see that the two tanks will reach 110 liters 10 minutes apart—tank B after 40 minutes of filling and tank A after 50 minutes of filling.

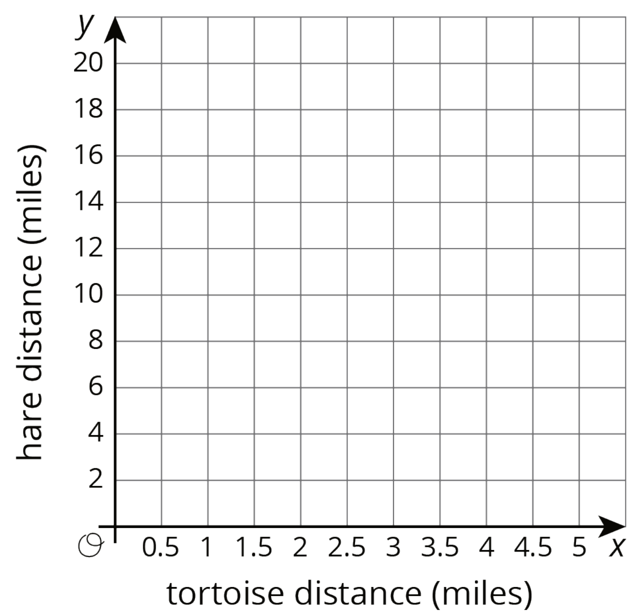
It is important to note that both of these graphs are correct, but one uses a range of values that helps answer the question. In order to always pick a helpful scale, we should consider the situation and the questions asked about it.

NAME DATE PERIOD

Unit 3, Lesson 2: Graphs of Proportional Relationships

1. The tortoise and the hare are having a race. After the hare runs 16 miles the tortoise has only run 4 miles.

   The relationship between the distance x the tortoise "runs" in miles for every y miles the hare runs is $y = 4x$. Graph this relationship.



2. The table shows a proportional relationship between the weight on a spring scale and the distance the spring has stretched.

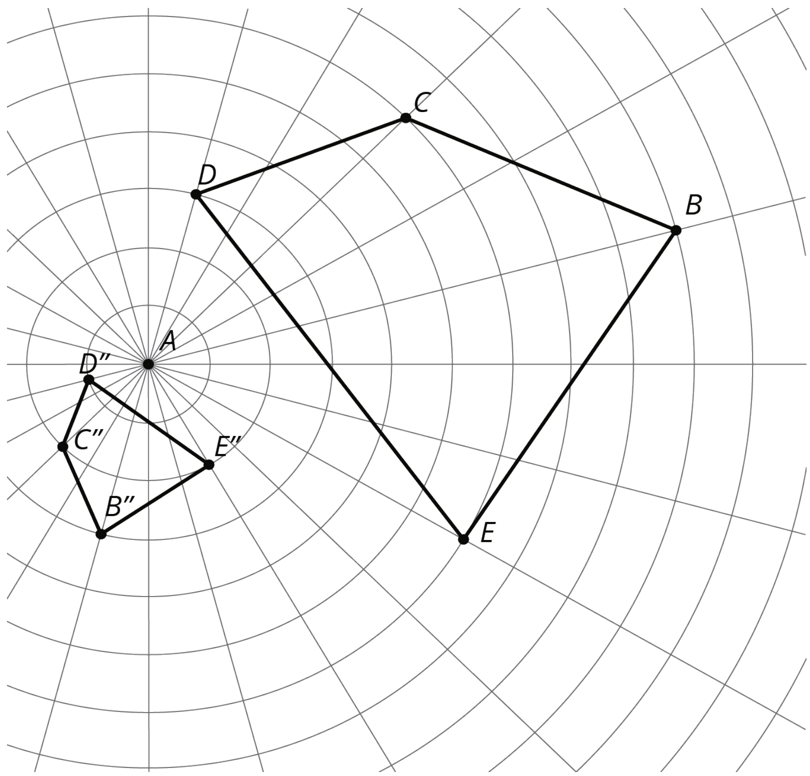
   a. Complete the table.

   b. Describe the scales you could use on the x and y axes of a coordinate grid that would show all the distances and weights in the table.

distance (cm)	weight (newtons)
20	28
55	
	140
1	

NAME DATE PERIOD

3. Find a sequence of rotations, reflections, translations, and dilations showing that one figure is similar to the other. Be specific: give the amount and direction of a translation, a line of reflection, the center and angle of a rotation, and the center and scale factor of a dilation.



(from Unit 2, Lesson 6)

4. Consider the following dialogue:

Andre said, “I found two figures that are congruent, so they can’t be similar.”

Diego said, “No, they are similar! The scale factor is 1.”

Who is correct? Use the definition of similarity to explain your answer.

(from Unit 2, Lesson 6)

NAME DATE PERIOD

Unit 3, Lesson 3: Representing Proportional Relationships

Let's graph proportional relationships.

3.1: Number Talk: Multiplication

Find the value of each product mentally.

$15 \cdot 2$

$15 \cdot 0.5$

$15 \cdot 0.25$

$15 \cdot (2.25)$

3.2: Representations of Proportional Relationships

1. Here are two ways to represent a situation.

   Description: Jada and Noah counted the number of steps they took to walk a set distance. To walk the same distance,

   - Jada took 8 steps
   - Noah took 10 steps

   Then they found that when Noah took 15 steps, Jada took 12 steps.

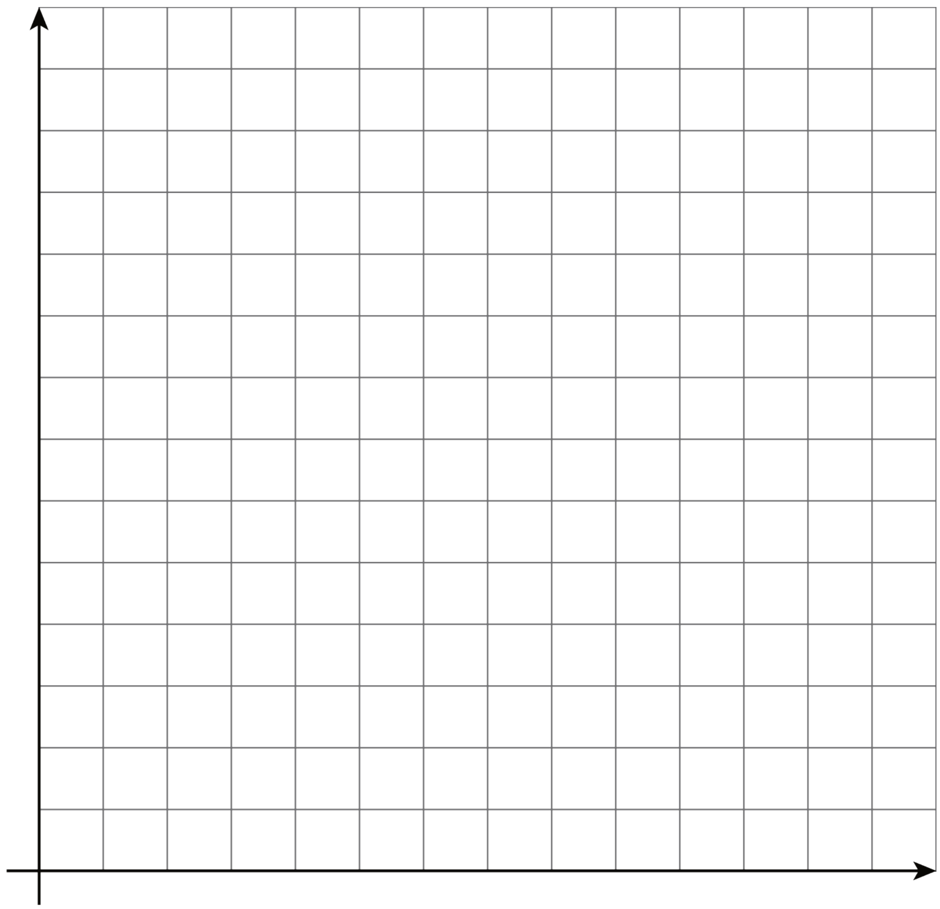
   Equation: Let x represent the number of steps Jada takes and let y represent the number of steps Noah takes.

   $$y = \frac{5}{4}x$$

   a. Create a table that represents this situation with at least 3 pairs of values.

NAME DATE PERIOD

b. Graph this relationship and label the axes.



c. How can you see or calculate the constant of proportionality in each representation? What does it mean?

d. Explain how you can tell that the equation, description, graph, and table all represent the same situation.

NAME DATE PERIOD

2. Here are two ways to represent a situation.

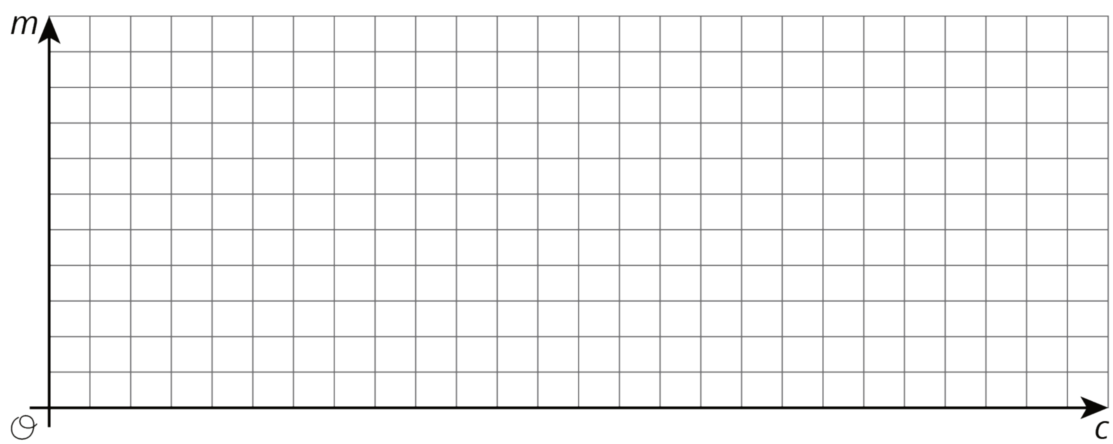
Description: The Origami Club is doing a car wash fundraiser to raise money for a trip. They charge the same price for every car. After 11 cars, they raised a total of $93.50. After 23 cars, they raised a total of $195.50.

Table:

number of cars	**amount raised in dollars**
11	93.50
23	195.50

a. Write an equation that represents this situation. (Use c to represent number of cars and use m to represent amount raised in dollars.)

b. Create a graph that represents this situation.



c. How can you see or calculate the constant of proportionality in each representation? What does it mean?

d. Explain how you can tell that the equation, description, graph, and table all represent the same situation.

NAME DATE PERIOD

3.3: Info Gap: Proportional Relationships

Your teacher will give you either a problem card or a data card. Do not show or read your card to your partner.

If your teacher gives you the *problem card*:

1. Silently read your card and think about what information you need to answer the question.
2. Ask your partner for the specific information that you need.
3. Explain to your partner how you are using the information to solve the problem.
4. Solve the problem and explain your reasoning to your partner.

If your teacher gives you the *data card*:

1. Silently read the information on your card.
2. Ask your partner “What specific information do you need?” and wait for your partner to *ask* for information. *Only* give information that is on your card. (Do not figure out anything for your partner!)
3. Before telling your partner the information, ask “Why do you need that information?”
4. After your partner solves the problem, ask them to explain their reasoning and listen to their explanation.

Pause here so your teacher can review your work. Ask your teacher for a new set of cards and repeat the activity, trading roles with your partner.

Are you ready for more?

Ten people can dig five holes in three hours. If n people digging at the same rate dig m holes in d hours:

1. Is n proportional to m when $d = 3$?
2. Is n proportional to d when $m = 5$?
3. Is m proportional to d when $n = 10$?

NAME DATE PERIOD

Lesson 3 Summary

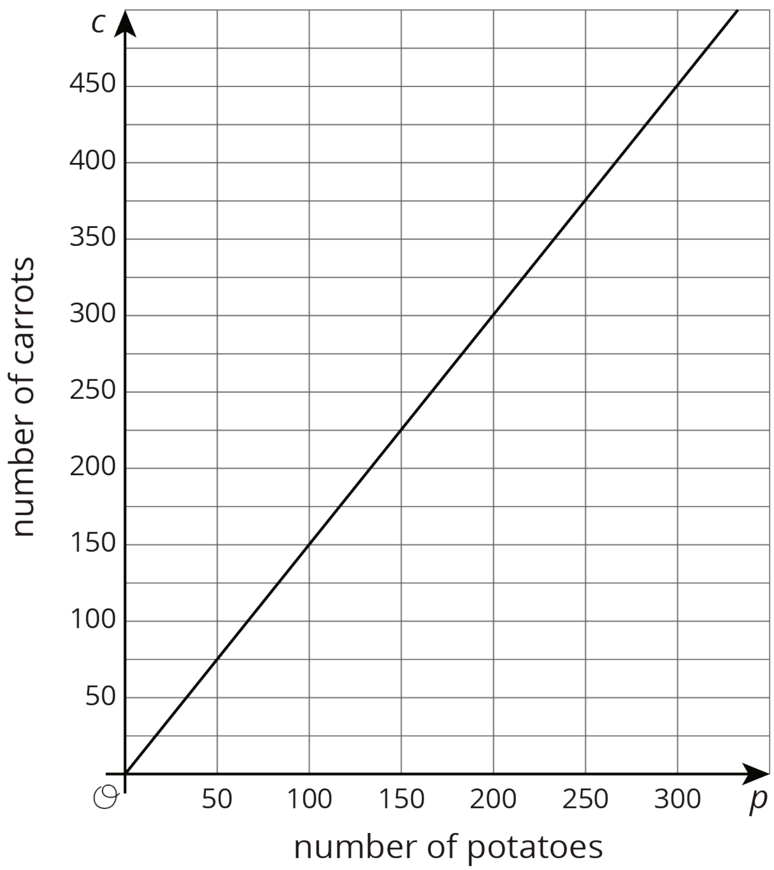
Proportional relationships can be represented in multiple ways. Which representation we choose depends on the purpose. And when we create representations we can choose helpful values by paying attention to the context. For example, a stew recipe calls for 3 carrots for every 2 potatoes. One way to represent this is using an equation. If there are p potatoes and c carrots, then $c = \frac{3}{2}p$.

Suppose we want to make a large batch of this recipe for a family gathering, using 150 potatoes. To find the number of carrots we could just use the equation: $\frac{3}{2} \boldsymbol{\cdot} 150 = 225$ carrots.

Now suppose the recipe is used in a restaurant that makes the stew in large batches of different sizes depending on how busy a day it is, using up to 300 potatoes at at time. Then we might make a graph to show how many carrots are needed for different amounts of potatoes. We set up a pair of coordinate axes with a scale from 0 to 300 along the horizontal axis and 0 to 450 on the vertical axis, because $450 = \frac{3}{2} \boldsymbol{\cdot} 300$. Then we can read how many carrots are needed for any number of potatoes up to 300.

NAME DATE PERIOD

Or if the recipe is used in a food factory that produces very large quantities and the potatoes come in bags of 150, we might just make a table of values showing the number of carrots needed for different multiplies of 150.



number of potatoes	number of carrots
150	225
300	450
450	675
600	900

No matter the representation or the scale used, the constant of proportionality, $\frac{3}{2}$, is evident in each. In the equation it is the number we multiply p by; in the graph, it is the slope; and in the table, it is the number we multiply values in the left column to get numbers in the right column. We can think of the constant of proportionality as a **rate of change** of c with respect to p. In this case the rate of change is $\frac{3}{2}$ carrots per potato.

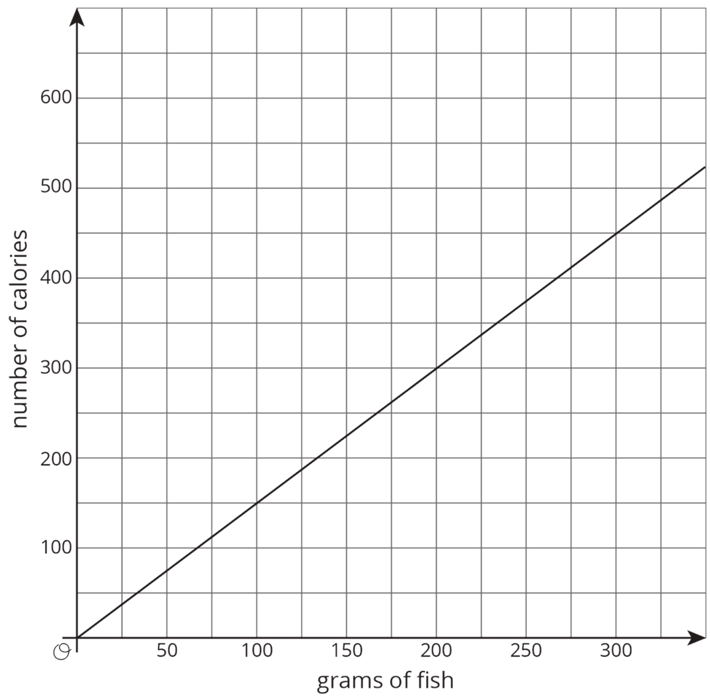
Lesson 3 Glossary Terms

- rate of change

NAME DATE PERIOD

Unit 3, Lesson 3: Representing Proportional Relationships

1. Here is a graph of the proportional relationship between calories and grams of fish:



a. Write an equation that reflects this relationship using x to represent the amount of fish in grams and y to represent the number of calories.

b. Use your equation to complete the table:

grams of fish	number of calories
1000	
	2001
1	

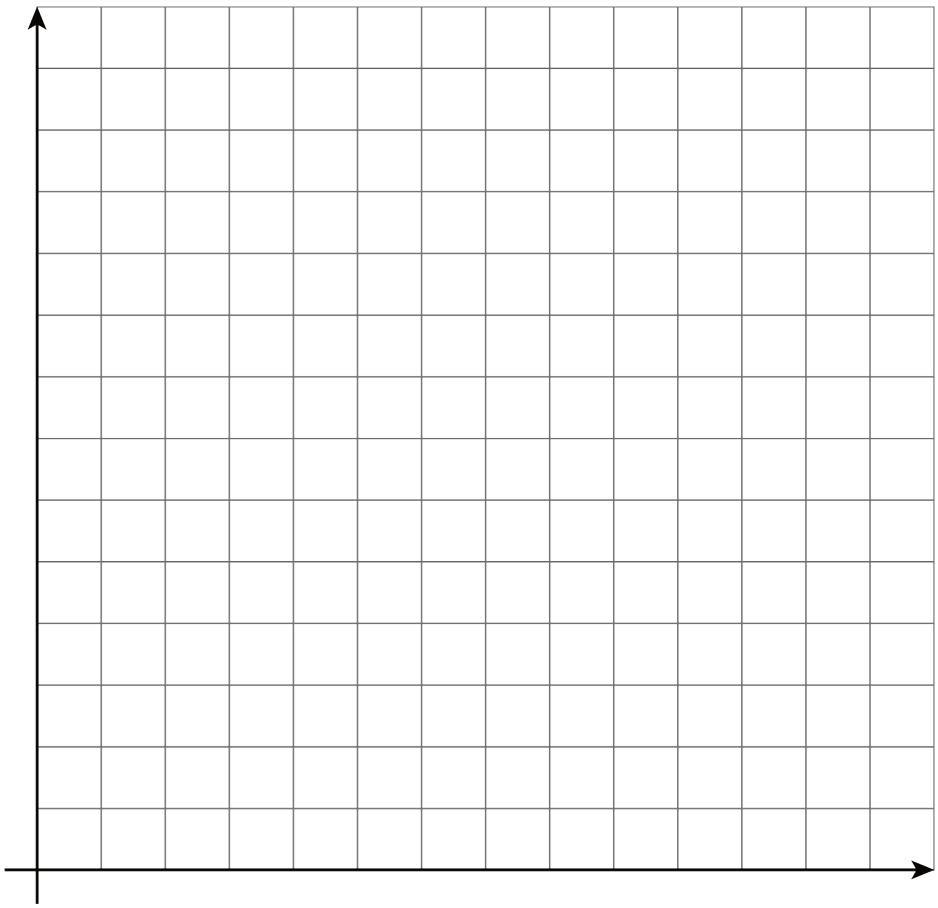
2. Students are selling raffle tickets for a school fundraiser. They collect $24 for every 10 raffle tickets they sell.

a. Suppose M is the amount of money the students collect for selling R raffle tickets. Write an

NAME DATE PERIOD

equation that reflects the relationship between M and R.

b. Label and scale the axes and graph this situation with M on the vertical axis and R on the horizontal axis. Make sure the scale is large enough to see how much they would raise if they sell 1000 tickets.



3. Describe how you can tell whether a line's slope is greater than 1, equal to 1, or less than 1.

   (from Unit 2, Lesson 10)

4. A line is represented by the equation $\frac{y}{x-2} = \frac{3}{11}$. What are the coordinates of some points that lie on the line? Graph the line on graph paper.

   (from Unit 2, Lesson 12)

NAME DATE PERIOD

Unit 3, Lesson 4: Comparing Proportional Relationships

Let’s compare proportional relationships.

4.1: What's the Relationship?

The equation $y = 4.2x$ could represent a variety of different situations.

1. Write a description of a situation represented by this equation. Decide what quantities x and y represent in your situation.

2. Make a table and a graph that represent the situation.

NAME DATE PERIOD

4.2: Comparing Two Different Representations

1. Elena babysits her neighbor's children. Her earnings are given by the equation $y = 8.40x$, where x represents the number of hours she worked and y represents the amount of money she earned. Jada earns $7 per hour mowing her neighbors' lawns.
   a. Who makes more money after working 12 hours? How much more do they make? Explain how you know.

   b. What is the rate of change for each situation and what does it mean?

   c. How long would it take each person to earn $150? Explain or show your reasoning.

2. Clare and Han have summer jobs stuffing envelopes for two different companies. Clare's earnings can be seen in the table.

   Han earns $15 for every 300 envelopes he finishes.

number of envelopes	money in dollars
400	40
900	90

   a. Who would make more money after stuffing 1500 envelopes? How much more money would they make? Explain how you know.

   b. What is the rate of change for each situation and what does it mean?

NAME DATE PERIOD

c. Who gets paid more in their job? Explain or show your reasoning.

3. Tyler plans to start a lemonade stand and is trying to perfect his recipe for lemonade. He wants to make sure the recipe doesn't use too much lemonade mix (lemon juice and sugar) but still tastes good.
   Lemonade Recipe 1 is given by the equation $y = 4x$ where x represents the amount of lemonade mix in cups and y represents the amount of water in cups.
   Lemonade Recipe 2 is given in the table.

lemonade mix (cups)	**water (cups)**
10	50
13	65
21	105

a. If Tyler had 16 cups of lemonade mix, how many cups of water would he need for each recipe? Explain how you know.

b. What is the rate of change for each situation and what does it mean?

c. Tyler has a 5-gallon jug (which holds 80 cups) to use for his lemonade stand and 16 cups of lemonade mix. Which lemonade recipe should he use? Explain or show your reasoning.

NAME DATE PERIOD

Are you ready for more?

Han and Clare are still stuffing envelopes. Han can stuff 20 envelopes in a minute, and Clare can stuff 10 envelopes in a minute. They start working together on a pile of 1,000 envelopes.

1. How long does it take them to finish the pile?

2. Who earns more money?

Lesson 4 Summary

When two proportional relationships are represented in different ways, we compare them by finding a common piece of information.

For example, Clare's earnings are represented by the equation $y = 14.5x$, where y is her earnings in dollars for working x hours.

The table shows some information about Jada's pay.

Who is paid at a higher rate per hour? How much more does that person have after 20 hours?

time worked (hours)	earnings (dollars)
7	92.75
4.5	59.63
37	490.25

In Clare's equation we see that the constant of proportionality relating her earnings to time worked is 14.50. This means that she earns $14.50 per hour.

We can calculate Jada's constant of proportionality by dividing a value in the earnings column by a value in the same row in the time worked column. Using the last row, the constant of proportionality for Jada is 13.25, since $490.25 \div 37 = 13.25$. An equation representing Jada's earnings is $y = 13.25x$. This means she earns $13.25 per hour.

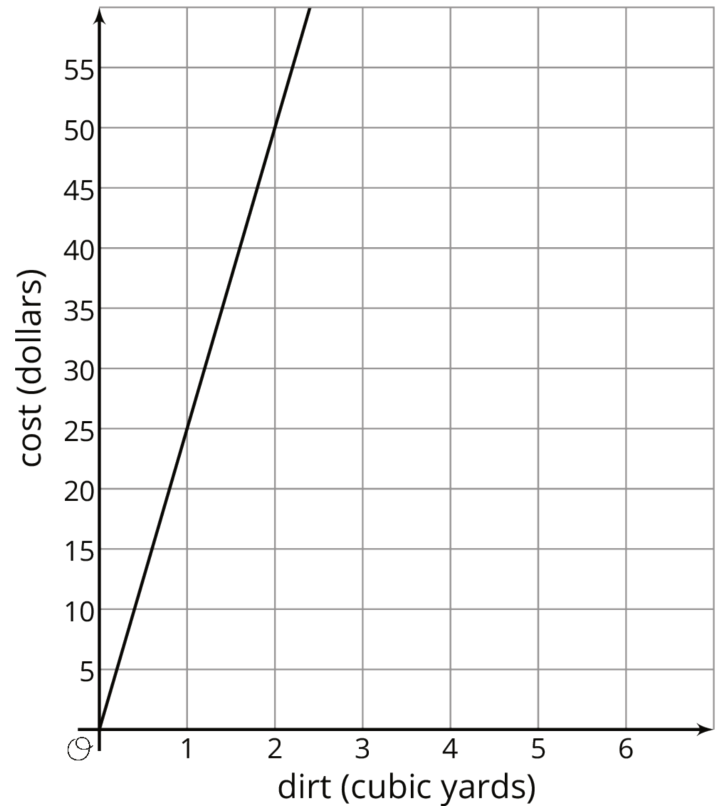
So Clare is paid at a higher rate than Jada. Clare earns $1.25 more per hour than Jada, which means that after 20 hours of work, she has $20 \cdot \$1.25 = \$25$ more than Jada.

NAME DATE PERIOD

Unit 3, Lesson 4: Comparing Proportional Relationships

1. A contractor must haul a large amount of dirt to a work site. She collected information from two hauling companies. EZ Excavation gives its prices in a table. Happy Hauling Service gives its prices in a graph.

dirt (cubic yards)	cost (dollars)
8	196
20	490
26	637



   a. How much would each hauling company charge to haul 40 cubic yards of dirt? Explain or show your reasoning.

   b. Calculate the rate of change for each relationship. What do they mean for each company?

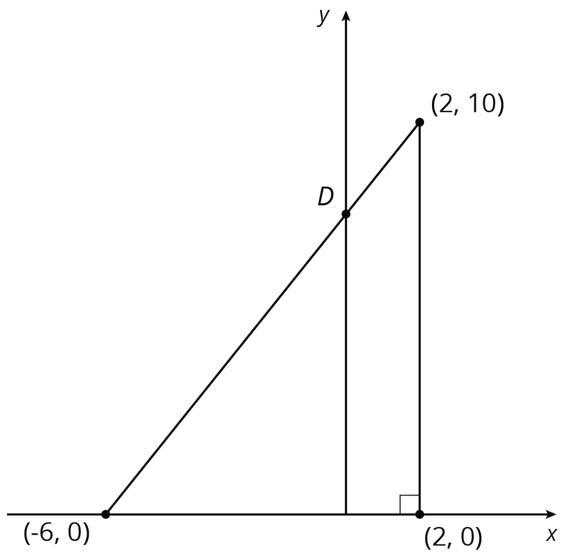
   c. If the contractor has 40 cubic yards of dirt to haul and a budget of $1000, which hauling company should she hire? Explain or show your reasoning.

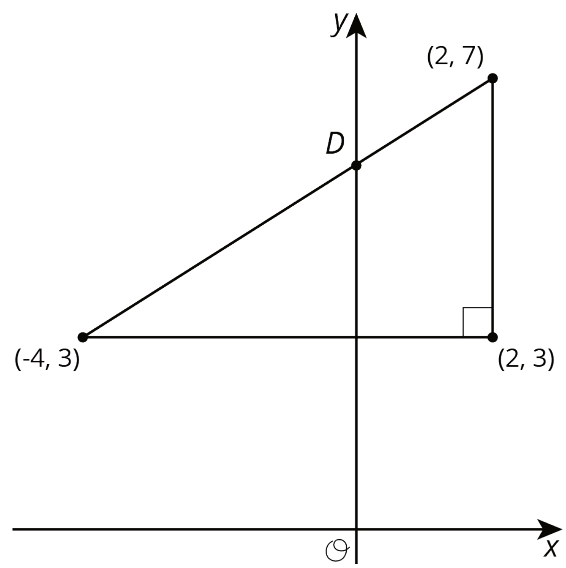
2. Andre and Priya are tracking the number of steps they walk. Andre records that he can walk 6000

NAME DATE PERIOD

steps in 50 minutes. Priya writes the equation $y = 118x$, where y is the number of steps and x is the number of minutes she walks, to describe her step rate. This week, Andre and Priya each walk for a total of 5 hours. Who walks more steps? How many more?

3. Find the coordinates of point D in each diagram:





(from Unit 2, Lesson 11)

4. Select **all** the pairs of points so that the line between those points has slope $\frac{2}{3}$.

A. $(0, 0)$ and $(2, 3)$
B. $(0, 0)$ and $(3, 2)$
C. $(1, 5)$ and $(4, 7)$
D. $(-2, -2)$ and $(4, 2)$
E. $(20, 30)$ and $(-20, -30)$

(from Unit 2, Lesson 11)

NAME DATE PERIOD

Unit 3, Lesson 5: Introduction to Linear Relationships

Let's explore some relationships between two variables.

5.1: Number Talk: Fraction Division

Find the value of $2\frac{5}{8} \div \frac{1}{2}$.

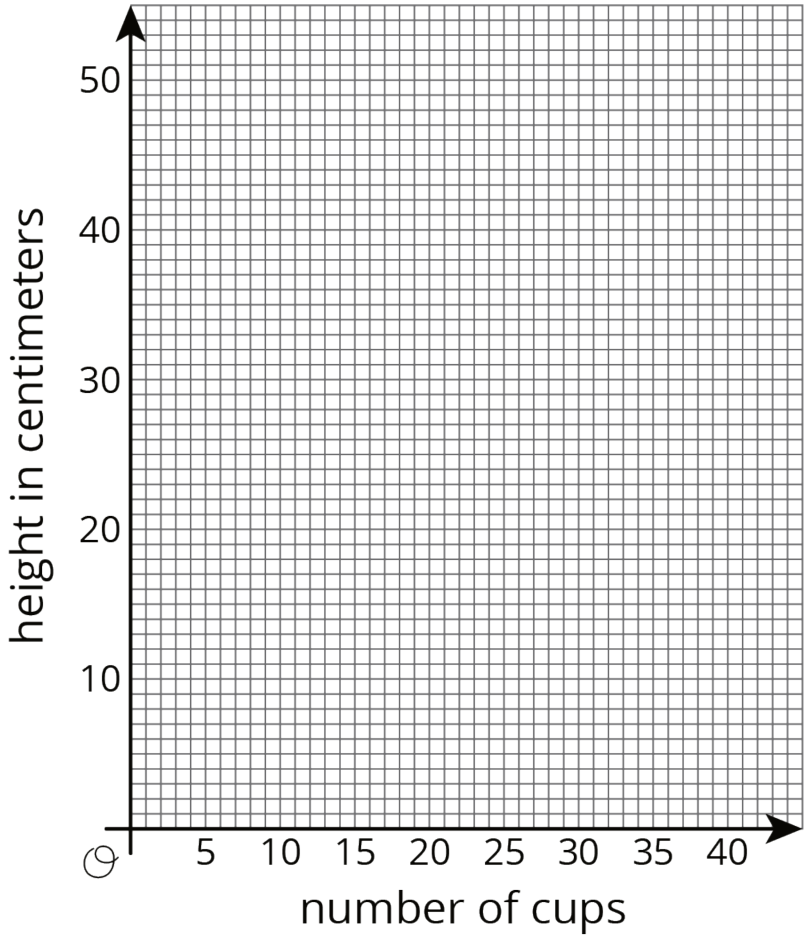
5.2: Stacking Cups

We have two stacks of styrofoam cups. One stack has 6 cups, and its height is 15 cm. The other one has 12 cups, and its height is 23 cm. How many cups are needed for a stack with a height of 50 cm?

NAME DATE PERIOD

NAME DATE PERIOD

5.3: Connecting Slope to Rate of Change



1. If you didn't create your own graph of the situation before, do so now.

2. What are some ways you can tell that the number of cups is not proportional to the height of the stack?

3. What is the **slope** of the line in your graph? What does the slope mean in this situation?

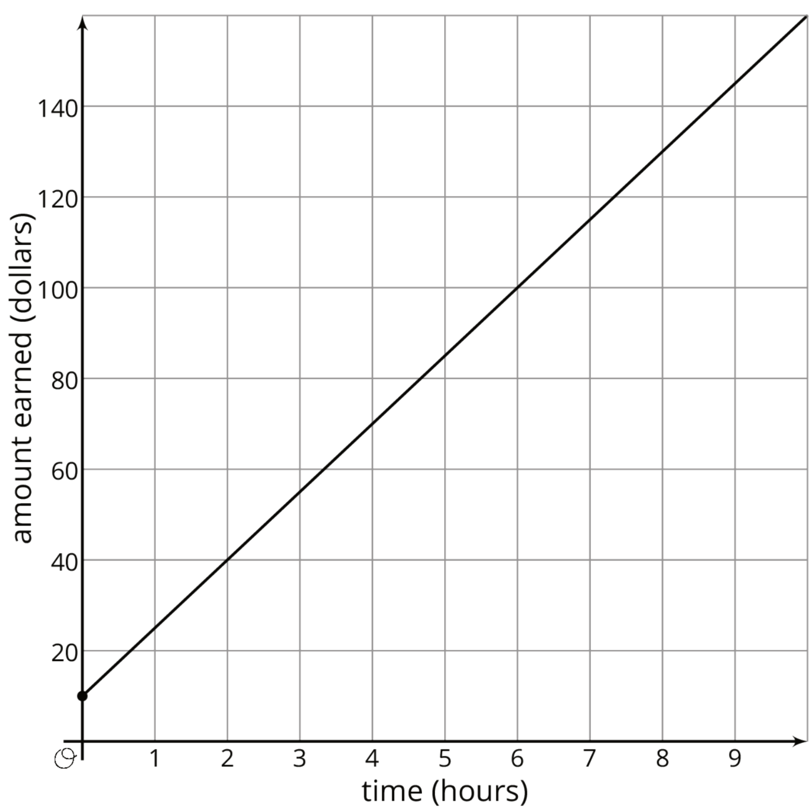
NAME DATE PERIOD

4. At what point does your line intersect the vertical axis? What do the coordinates of this point tell you about the cups?

5. How much height does each cup after the first add to the stack?

Lesson 5 Summary

Andre starts babysitting and charges \$10 for traveling to and from the job, and \$15 per hour. For every additional hour he works he charges another \$15. If we graph Andre's earnings based on how long he works, we have a line that starts at \$10 on the vertical axis and then increases by \$15 each hour. A **linear relationship** is any relationship between two quantities where one quantity has a constant **rate of change** with respect to the other.



We can figure out the rate of change using the graph. Because the rate of change is constant, we can take any two points on the graph and divide the amount of vertical change by the amount of horizontal change. For example, take the points $(2,40)$ and $(6,100)$. They mean that Andre earns $40 for working 2 hours and $100 for working 6 hours. The rate of change is $\frac{100-40}{6-2}=15$ dollars per hour. Andre's earnings go up $15 for each hour of babysitting. Notice that this is the same way we calculate the **slope** of the line. That's why the graph is a line, and why we call this a linear relationship. The rate of change of a linear relationship is the same as the slope of its graph.

With proportional relationships we are used to graphs that contain the point $(0,0)$. But proportional relationships are just one type of linear relationship. In the following lessons, we will continue to explore the other type of linear relationship where the quantities are not both 0 at the same time.

Lesson 5 Glossary Terms

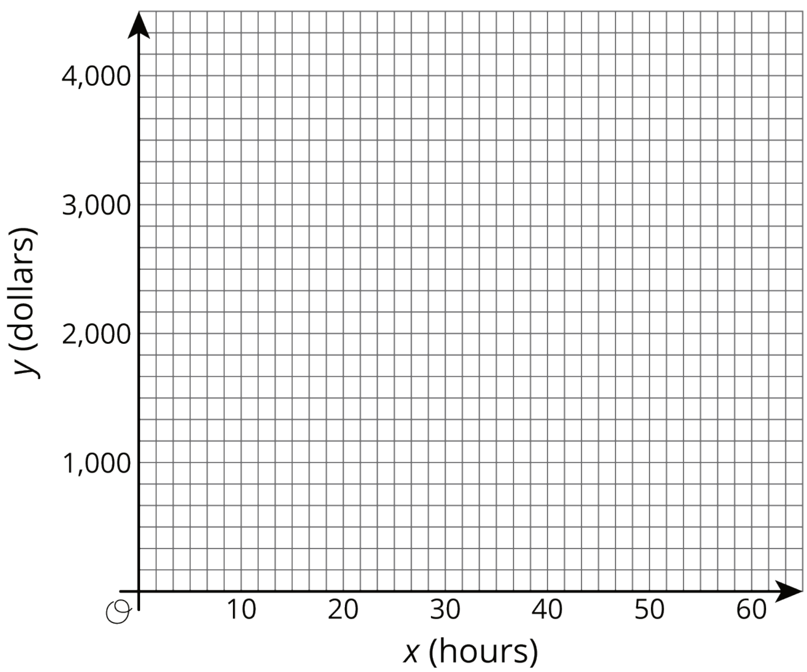
- slope
- rate of change
- linear relationship

NAME DATE PERIOD

Unit 3, Lesson 5: Introduction to Linear Relationships

1. A restaurant offers delivery for their pizzas. The total cost is a delivery fee added to the price of the pizzas. One customer pays \$25 to have 2 pizzas delivered. Another customer pays \$58 for 5 pizzas. How many pizzas are delivered to a customer who pays \$80?

2. To paint a house, a painting company charges a flat rate of \$500 for supplies, plus \$50 for each hour of labor.



   a. How much would the painting company charge to paint a house that needs 20 hours of labor? A house that needs 50 hours?

   b. Draw a line representing the relationship between x, the number of hours it takes the painting company to finish the house, and y, the total cost of painting the house. Label the two points from the earlier question on your graph.

   c. Find the slope of the line. What is the meaning of the slope in this context?

3. Tyler and Elena are on the cross country team.

   Tyler's distances and times for a training run are shown on the graph.

NAME DATE PERIOD

Elena's distances and times for a training run are given by the equation $y = 8.5x$, where x represents distance in miles and y represents time in minutes.

a. Who ran farther in 10 minutes? How much farther? Explain how you know.

b. Calculate each runner's pace in minutes per mile.

c. Who ran faster during the training run? Explain or show your reasoning.

(from Unit 3, Lesson 4)

NAME DATE PERIOD

4. Write an equation for the line that passes through $(2, 5)$ and $(6, 7)$.

   (from Unit 2, Lesson 12)

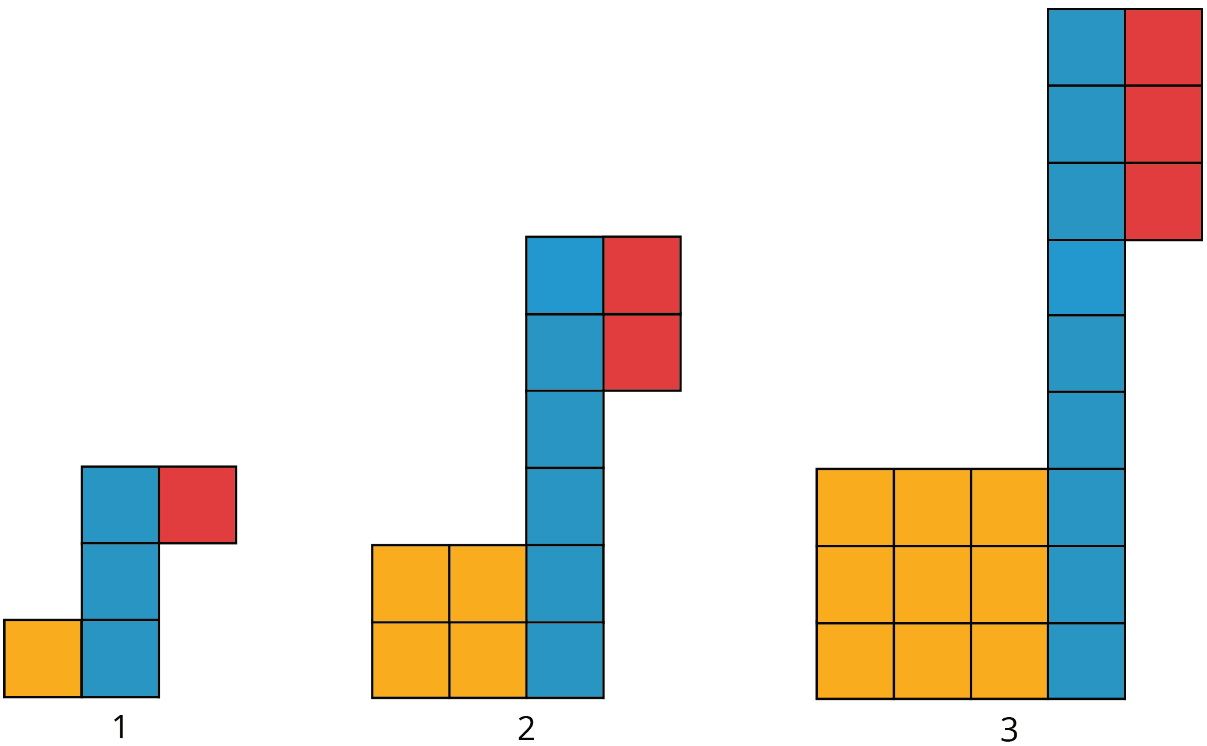
NAME DATE PERIOD

Unit 3, Lesson 6: More Linear Relationships

Let's explore some more relationships between two variables.

6.1: Growing

Look for a growing pattern. Describe the pattern you see.



1. If your pattern continues growing in the same way, how many tiles of each color will be in the 4th and 5th diagram? The 10th diagram?

2. How many tiles of each color will be in the nth diagram? Be prepared to explain how your reasoning.

NAME DATE PERIOD

6.2: Slopes, Vertical Intercepts, and Graphs

Your teacher will give you 6 cards describing different situations and 6 cards with graphs.

1. Match each situation to a graph.

2. Pick one proportional relationship and one non-proportional relationship and answer the following questions about them.
   a. How can you find the slope from the graph? Explain or show your reasoning.

   b. Explain what the slope means in the situation.

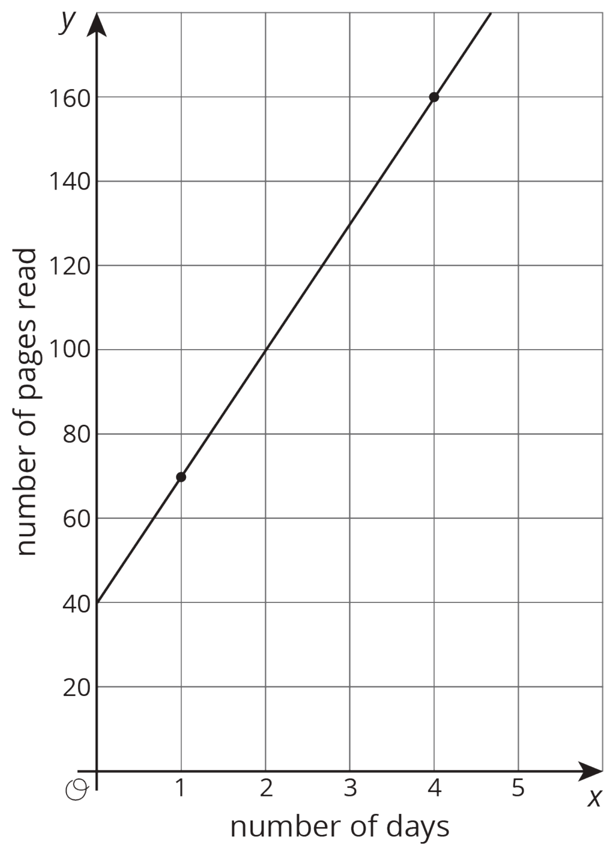
   c. Find the point where the line crosses the vertical axis. What does that point tell you about the situation?

NAME DATE PERIOD

6.3: Summer Reading

Lin has a summer reading assignment. After reading the first 30 pages of the book, she plans to read 40 pages each day until she finishes. Lin makes the graph shown here to track how many total pages she'll read over the next few days.

After day 1, Lin reaches page 70, which matches the point $(1, 70)$ she made on her graph. After day 4, Lin reaches page 190, which does not match the point $(4, 160)$ she made on her graph. Lin is not sure what went wrong since she knows she followed her reading plan.



1. Sketch a line showing Lin's original plan on the axes.

2. What does the **vertical intercept** mean in this situation? How do the vertical intercepts of the two lines compare?

3. What does the slope mean in this situation? How do the slopes of the two lines compare?

NAME DATE PERIOD

Are you ready for more?

Jada's grandparents started a savings account for her in 2010. The table shows the amount in the account each year.

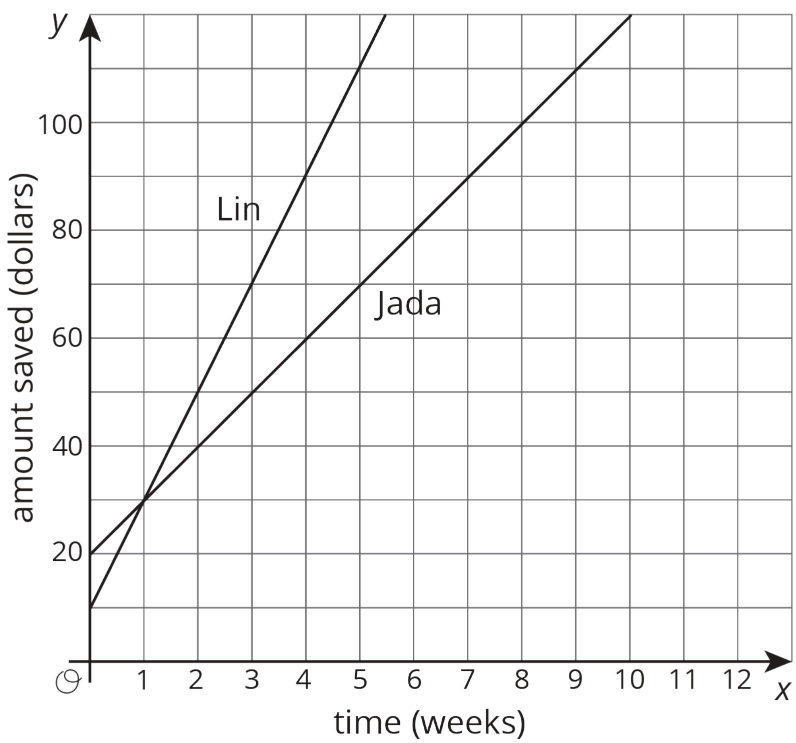
year	amount in dollars
2010	600
2012	750
2014	900
2016	1050

If this relationship is graphed with the year on the horizontal axis and the amount in dollars on the vertical axis, what is the vertical intercept? What does it mean in this context?

NAME DATE PERIOD

Lesson 6 Summary

At the start of summer break, Jada and Lin decide to save some of the money they earn helping out their neighbors to use during the school year. Jada starts by putting \$20 into a savings jar in her room and plans to save \$10 a week. Lin starts by putting \$10 into a savings jar in her room plans to save \$20 a week. Here are graphs of how much money they will save after 10 weeks if they each follow their plans:



The value where a line intersects the vertical axis is called the **vertical intercept**. When the vertical axis is labeled with a variable like y, this value is also often called the y-*intercept*. Jada's graph has a vertical intercept of \$20 while Lin's graph has a vertical intercept of \$10. These values reflect the amount of money they each started with. At 1 week they will have saved the same amount, \$30. But after week 1, Lin is saving more money per week (so she has a larger rate of change), so she will end up saving more money over the summer if they each follow their plans.

Lesson 6 Glossary Terms

- vertical intercept

NAME DATE PERIOD

Unit 3, Lesson 6: More Linear Relationships

1. Explain what the slope and intercept mean in each situation.

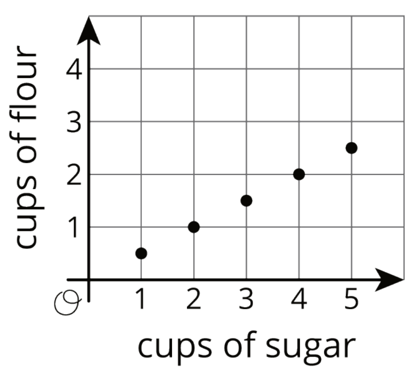
   a. A graph represents the perimeter, y, in units, for an equilateral triangle with side length x units. The slope of the line is 3 and the y-intercept is 0.

   b. The amount of money, y, in a cash box after x tickets are purchased for carnival games. The slope of the line is $\frac{1}{4}$ and the y-intercept is 8.

   c. The number of chapters read, y, after x days. The slope of the line is $\frac{5}{4}$ and the y-intercept is 2.

   d. The graph shows the cost in dollars, y, of a muffin delivery and the number of muffins, x, ordered. The slope of the line is 2 and the y-intercept is 3.

2. The graph shows the relationship between the number of cups of flour and the number of cups of sugar in Lin's favorite brownie recipe.



The table shows the amounts of flour and sugar needed for Noah's favorite brownie recipe.

NAME DATE PERIOD

amount of sugar (cups)	amount of flour (cups)
$\frac{3}{2}$	1
3	2
$4\frac{1}{2}$	3

a. Noah and Lin buy a 12-cup bag of sugar and divide it evenly to make their recipes. If they each use all their sugar, how much flour do they each need?

b. Noah and Lin buy a 10-cup bag of flour and divide it evenly to make their recipes. If they each use all their flour, how much sugar do they each need?

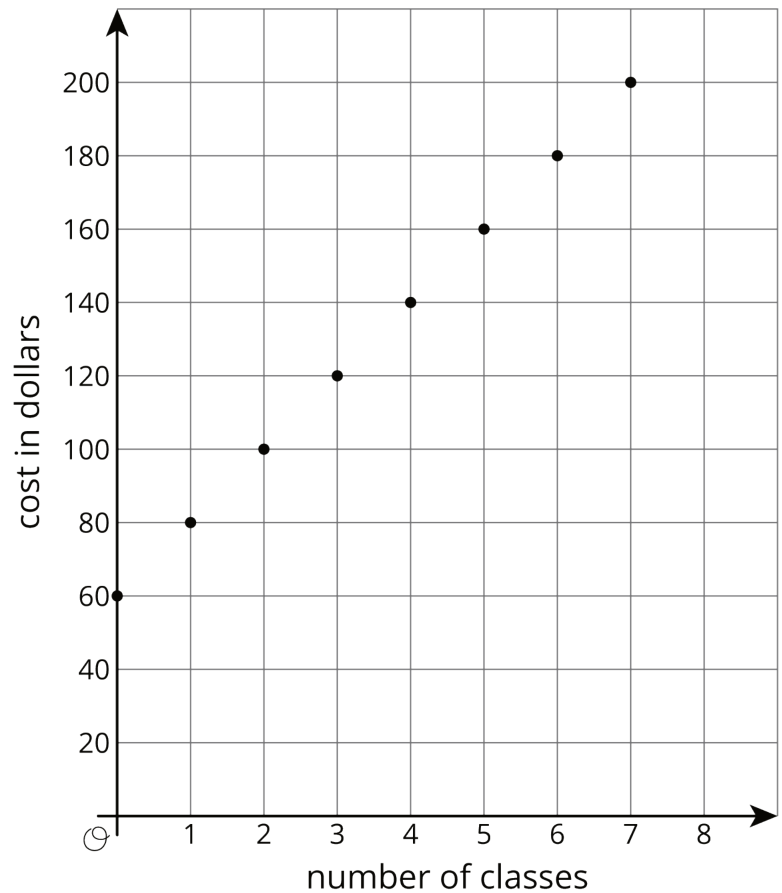
(from Unit 3, Lesson 4)

3. Customers at the gym pay a membership fee to join and then a fee for each class they attend. Here is a graph that represents the situation.

NAME DATE PERIOD

a. What does the slope of the line shown by the points mean in this situation?

b. What does the vertical intercept mean in this situation?



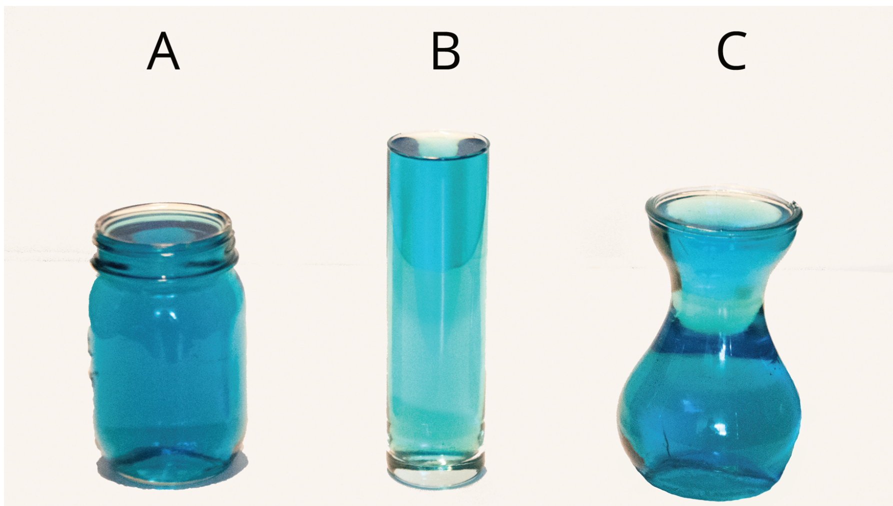
NAME DATE PERIOD

Unit 3, Lesson 7: Representations of Linear Relationships

Let’s write equations from real situations.

7.1: Estimation: Which Holds More?

Which glass will hold the most water? The least?



7.2: Rising Water Levels

m.openup.org/1/8-3-7-2

1. Record data from your teacher's demonstration in the table. (You may not need all the rows.)

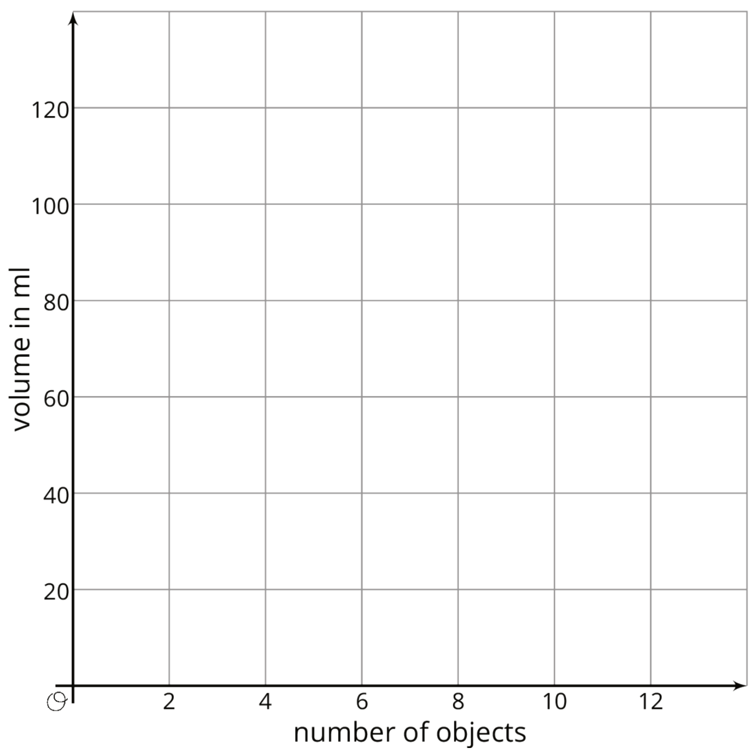
number of objects	volume in ml

NAME DATE PERIOD

2. What is the volume, V, in the cylinder after you add x objects? Explain your reasoning.

3. If you wanted to make the water reach the highest mark on the cylinder, how many objects would you need?

4. Plot and label points that show your measurements from the experiment.



5. The points should fall on a line. Use a ruler to graph this line.

NAME DATE PERIOD

6. Compute the slope of the line. What does the slope mean in this situation?

7. What is the vertical intercept? What does vertical intercept mean in this situation?

Are you ready for more?

A situation is represented by the equation $y = 5 + \frac{1}{2}x$.

1. Invent a story for this situation.

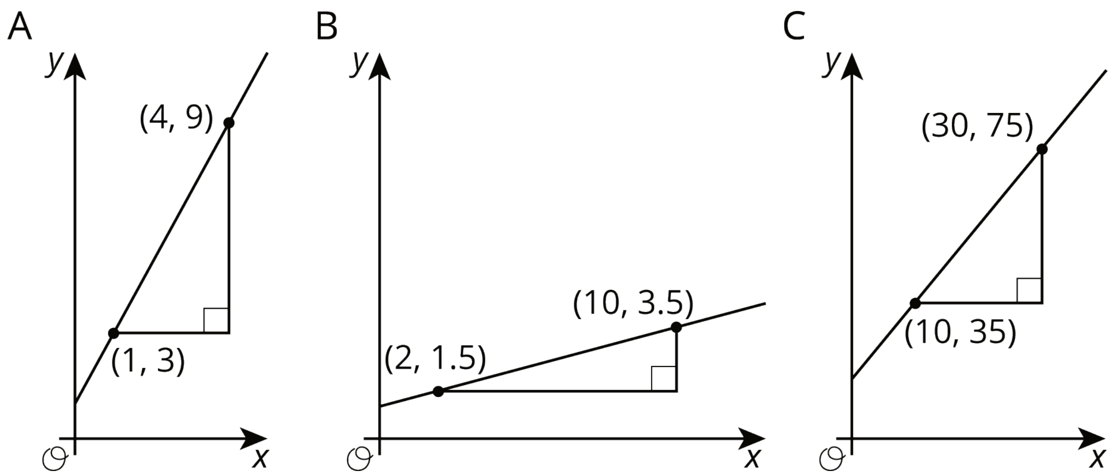
2. Graph the equation.

3. What do the $\frac{1}{2}$ and the 5 represent in your situation?

4. Where do you see the $\frac{1}{2}$ and 5 on the graph?

NAME DATE PERIOD

7.3: Calculate the Slope



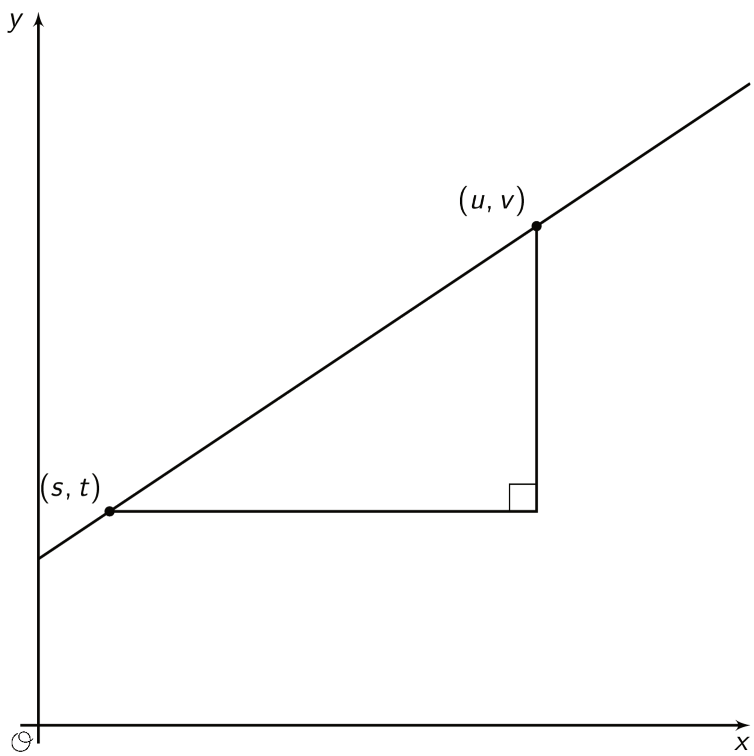
1. For each graph, record:

vertical change	horizontal change	slope

2. Describe a procedure for finding the slope between any two points on a line.

NAME DATE PERIOD

3. Write an expression for the slope of the line in the graph using the letters u, v, s, and t.



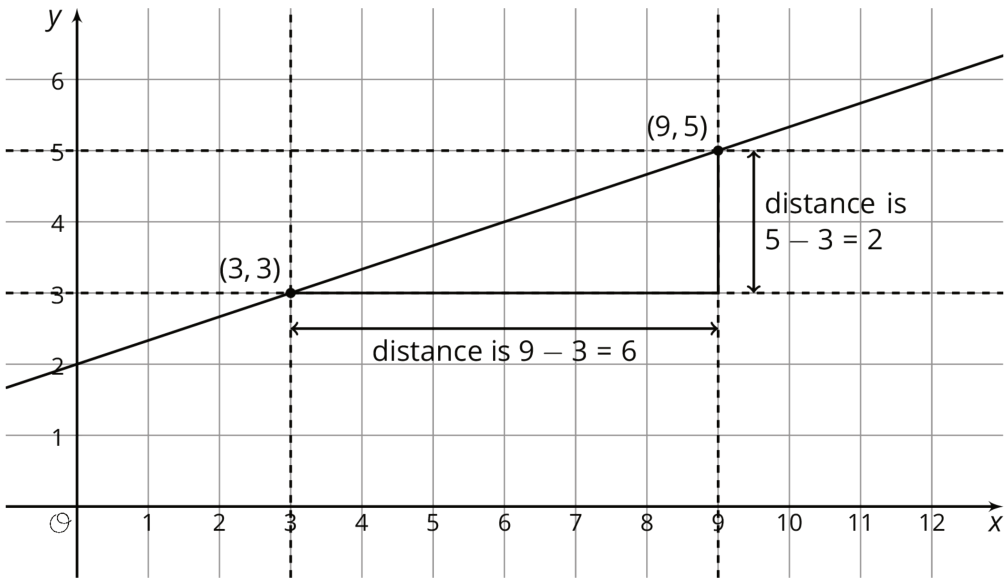
Lesson 7 Summary

Let's say we have a glass cylinder filled with 50 ml of water and a bunch of marbles that are 3 ml in volume. If we drop marbles into the cylinder one at a time, we can watch the height of the water increase by the same amount, 3 ml, for each one added. This constant rate of change means there is a linear relationship between the number of marbles and the height of the water. Add one marble, the water height goes up 3 ml. Add 2 marbles, the water height goes up 6 ml. Add x marbles, the water height goes up $3x$ ml.

Reasoning this way, we can calculate that the height, y, of the water for x marbles is $y = 3x + 50$. Any linear relationships can be expressed in the form $y = mx + b$ using just the rate of change, m, and the initial amount, b. The 3 represents the rate of change, or slope of the graph, and the 50 represents the initial amount, or vertical intercept of the graph. We'll learn about some more ways to think about this equation in future lessons.

NAME DATE PERIOD

Now what if we didn't have a description to use to figure out the slope and the vertical intercept? That's okay so long as we can find some points on the line! For the line graphed here, two of the points on the line are $(3, 3)$ and $(9, 5)$ and we can use these points to draw in a slope triangle as shown:



The slope of this line is the quotient of the length of the vertical side of the slope triangle and the length of the horizontal side of the slope triangle. So the slope, m, is $\frac{\text{vertical change}}{\text{horizontal change}} = \frac{2}{6} = \frac{1}{3}$. We can also see from the graph that the vertical intercept, b, is 2. Putting these together, we can say that the equation for this line is $y = \frac{1}{3}x + 2$.

NAME DATE PERIOD

Unit 3, Lesson 7: Representations of Linear Relationships

1. Here are recipes for two different banana cakes. Information for the first recipe is shown in the table.

sugar (cups)	flour (cups)
$\frac{1}{2}$	$\frac{3}{4}$
$2\frac{1}{2}$	$3\frac{3}{4}$
3	$4\frac{1}{2}$

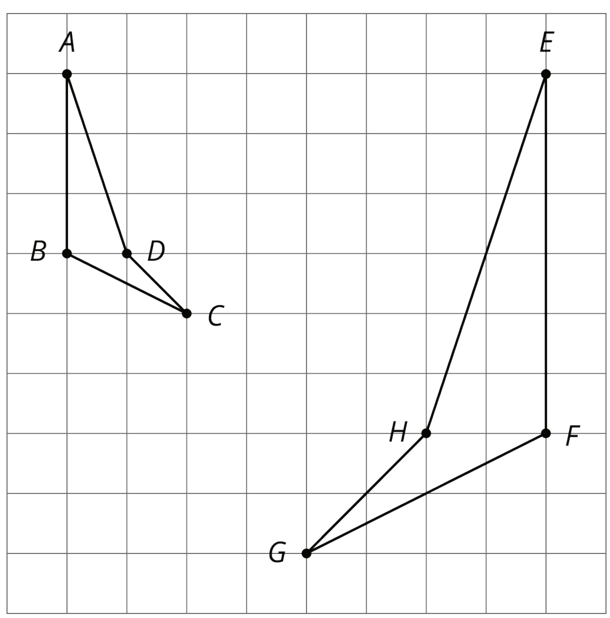
The relationship between cups of flour y and cups of sugar x in the second recipe is $y = \frac{7}{4}x$

a. If you used 4 cups of sugar, how much flour does each recipe need?

b. What is the constant of proportionality for each situation and what does it mean?

(from Unit 3, Lesson 4)

2. Show that the two figures are similar by identifying a sequence of translations, rotations, reflections, and dilations that takes the larger figure to the smaller one.



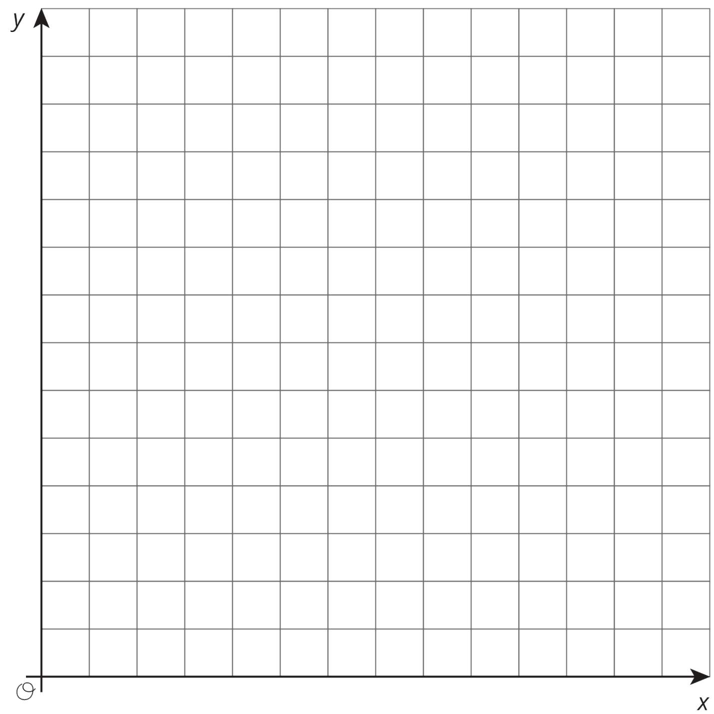
NAME DATE PERIOD

(from Unit 2, Lesson 6)

3. Create a graph that shows three linear relationships with different y-intercepts using the following slopes, and write an equation for each line.

   Slopes:

   - $\frac{1}{5}$
   - $\frac{3}{5}$
   - $\frac{6}{5}$

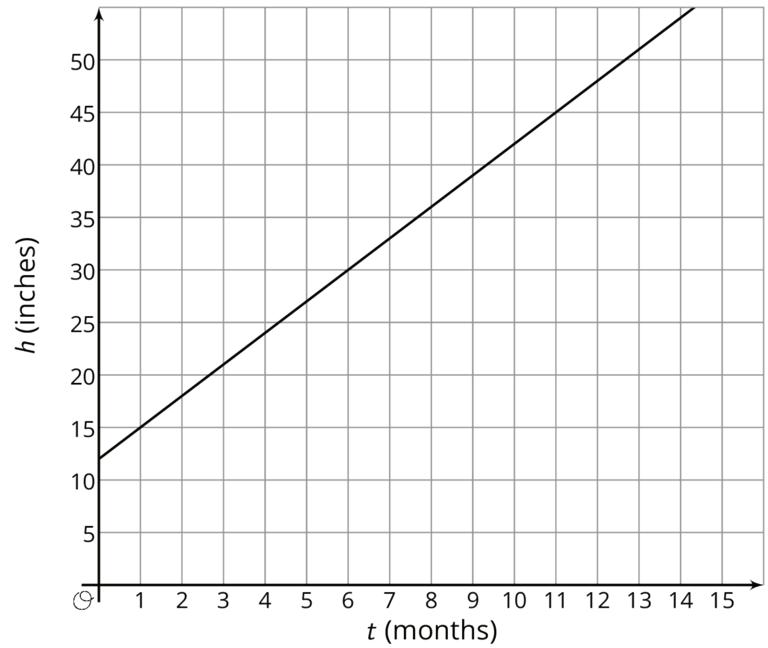


4. The graph shows the height in inches, h, of a bamboo plant t months after it has been planted.

NAME DATE PERIOD

a. Write an equation that describes the relationship between h and t.

b. After how many months will the bamboo plant be 66 inches tall? Explain or show your reasoning.

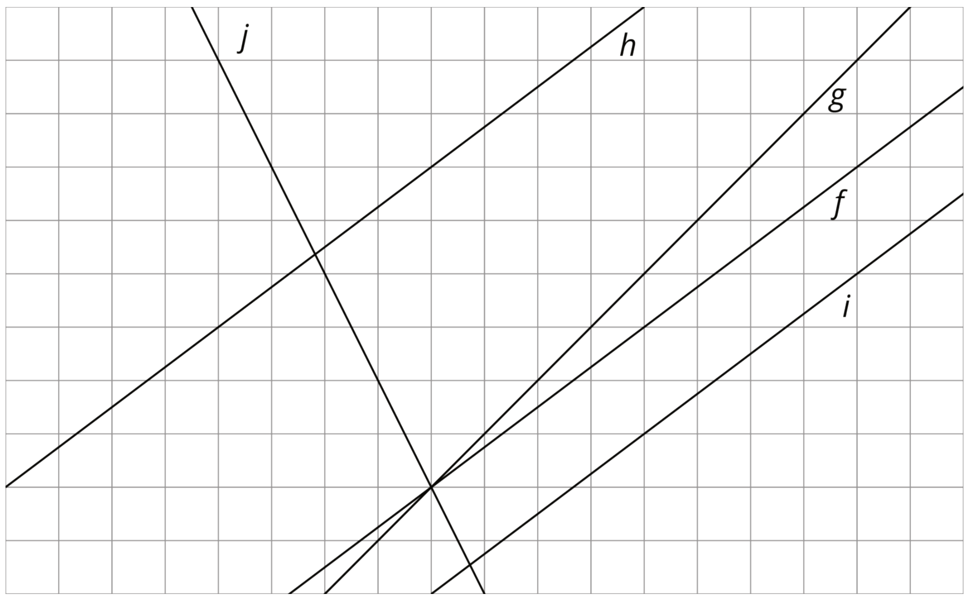


NAME DATE PERIOD

Unit 3, Lesson 8: Translating to $y = mx + b$

Let's see what happens to the equations of translated lines.

8.1: Lines that Are Translations



The diagram shows several lines. You can only see part of the lines, but they actually continue forever in both directions.

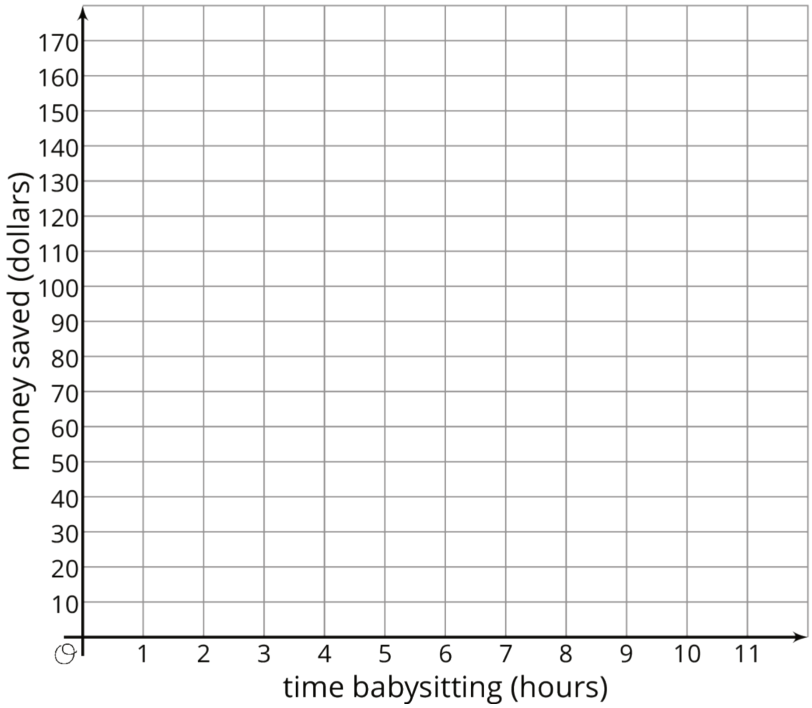
1. Which lines are images of line f under a translation?

2. For each line that is a translation of f, draw an arrow on the grid that shows the vertical translation distance.

NAME DATE PERIOD

8.2: Increased Savings

m.openup.org/1/8-3-8-2



1. Diego earns $10 per hour babysitting. Assume that he has no money saved before he starts babysitting and plans to save all of his earnings. Graph how much money, y, he has after x hours of babysitting.

2. Now imagine that Diego started with $30 saved before he starts babysitting. On the same set of axes, graph how much money, y, he would have after x hours of babysitting.

3. Compare the second line with the first line. How much *more* money does Diego have after 1 hour of babysitting? 2 hours? 5 hours? x hours?

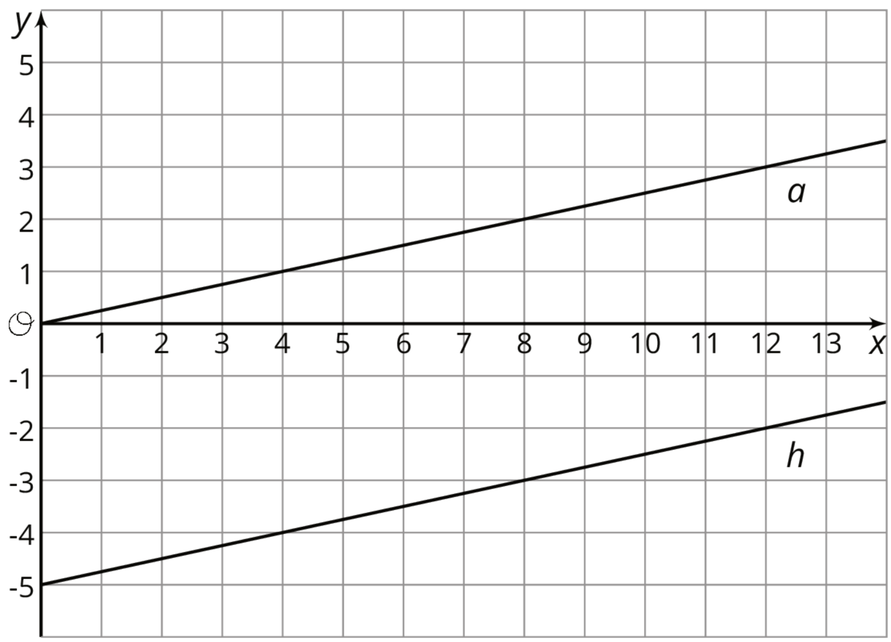
4. Write an equation for each line.

NAME DATE PERIOD

8.3: Translating a Line

m.openup.org/1/8-3-8-3

This graph shows two lines. Line a goes through the origin $(0, 0)$. Line h is the image of line a under a translation.



1. Select all of the equations whose graph is the line h.
   a. $y = \frac{1}{4}x - 5$
   b. $y = \frac{1}{4}x + 5$
   c. $\frac{1}{4}x - 5 = y$
   d. $y = \text{-}5 + \frac{1}{4}x$
   e. $\text{-}5 + \frac{1}{4}x = y$
   f. $y = 5 - \frac{1}{4}x$

2. Your teacher will give you 12 cards. There are 4 pairs of lines, A–D, showing the graph, a, of a proportional relationship and the image, h, of a under a translation. Match each line h with an equation and either a table or description. For the line with no matching equation, write one on the blank card.

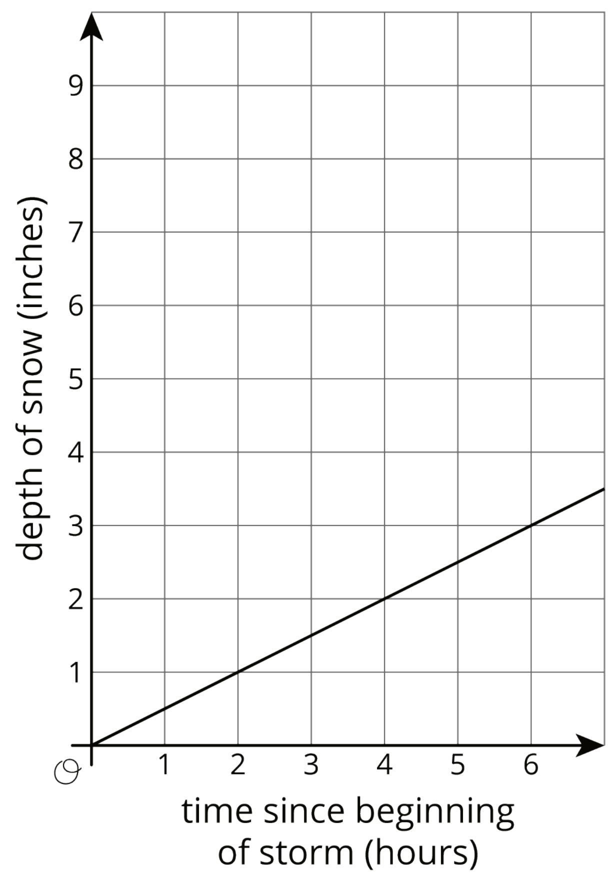
Are you ready for more?

A student says that the graph of the equation $y = 3(x + 8)$ is the same as the graph of $y = 3x$, only translated upwards by 8 units. Do you agree? Why or why not?

NAME DATE PERIOD

Lesson 8 Summary

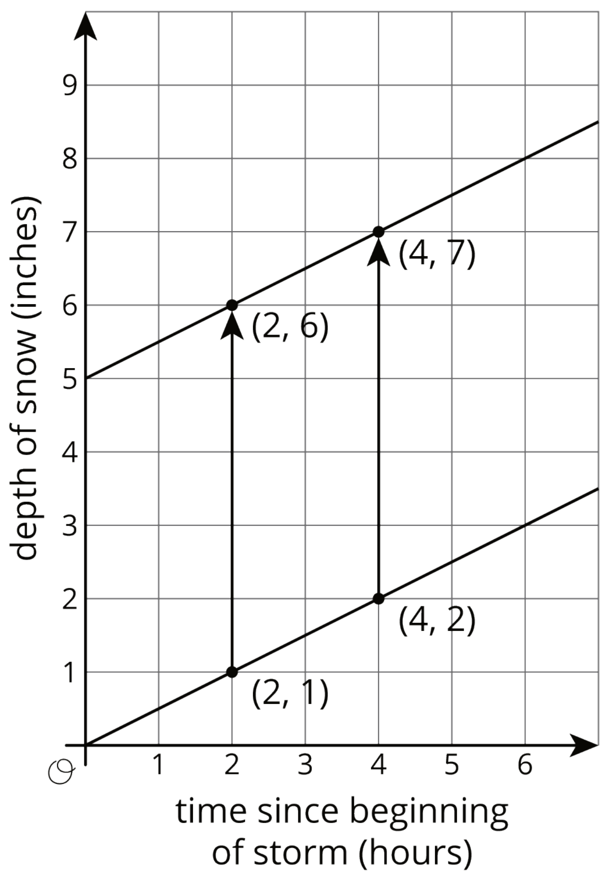
During an early winter storm, the snow fell at a rate of $\frac{1}{2}$ inches per hour. We can see the rate of change, $\frac{1}{2}$, in both the equation that represents this storm, $y = \frac{1}{2}x$, and in the slope of the line representing this storm.



In addition to being a linear relationship between the time since the beginning of the storm and the depth of the snow, we can also call this as a proportional relationship since the depth of snow was 0 at the beginning of the storm.

During a mid-winter storm, the snow again fell at a rate of $\frac{1}{2}$ inches per hour, but this time there was already 5 inches of snow on the ground. We can graph this storm on the same axes as the first storm by taking all the points on the graph of the first storm and translating them up 5 inches.

NAME DATE PERIOD



2 hours after each storm begins, 1 inch of new snow has fallen. For the first storm, this means there is now 1 inch of snow on the ground. For the second storm, this means there are now 6 inches of snow on the ground. Unlike the first storm, the second is not a proportional relationship since the line representing the second storm has a vertical intercept of 5. The equation representing the storm, $y = \frac{1}{2}x + 5$, is of the form $y = mx + b$, where m is the rate of change, also the slope of the graph, and b is the initial amount, also the vertical intercept of the graph.

NAME DATE PERIOD

Unit 3, Lesson 8: Translating to $y = mx + b$

1. Select **all** equations that have graphs with the same y-intercept.

   A. $y = 3x - 8$
   B. $y = 3x - 9$
   C. $y = 3x + 8$
   D. $y = 5x - 8$
   E. $y = 2x - 8$
   F. $y = \frac{1}{3}x - 8$

2. Create a graph showing the equations $y = \frac{1}{4}x$ and $y = \frac{1}{4}x - 5$. Explain how the graphs are the same and how they are different.

3. A cable company charges $70 per month for cable service to existing customers.

   a. Find a linear equation representing the relationship between x, the number of months of service, and y, the total amount paid in dollars by an existing customer.

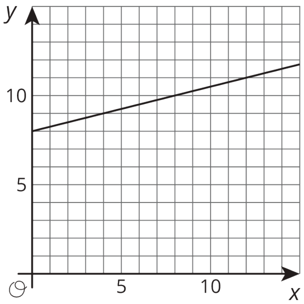
   b. For new customers, there is an additional one-time $100 service fee. Repeat the previous problem for new customers.

   c. When the two equations are graphed in the coordinate plane, how are they related to each other geometrically?

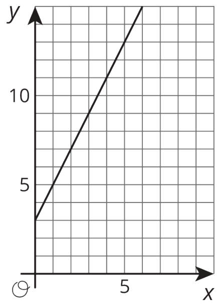
4. Match each graph to a situation.

NAME DATE PERIOD

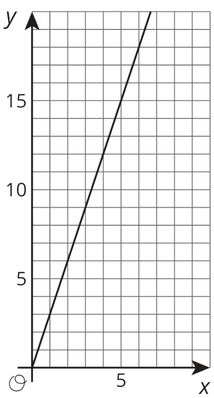
A



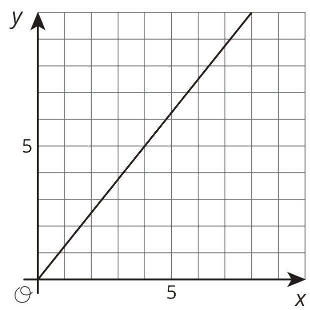
B



C



D



1. The graph represents the perimeter, y, in units, for an equilateral triangle with side length of x units. The slope of the line is 3.

2. The amount of money, y, in a cash box after x tickets are purchased for carnival games. The slope of the line is $\frac{1}{4}$.

3. The number of chapters read, y, after x days. The slope of the line is $\frac{5}{4}$.

4. The graph shows the cost in dollars, y, of a muffin delivery and the number of muffins, x, ordered. The slope of the line is 2.

(from Unit 3, Lesson 6)

5. A mountain road is 5 miles long and gains elevation at a constant rate. After 2 miles, the elevation is 5500 feet above sea level. After 4 miles, the elevation is 6200 feet above sea level.

   a. Find the elevation of the road at the point where the road begins.

   b. Describe where you would see the point in part (a) on a graph where y represents the elevation in feet and x represents the distance along the road in miles.

(from Unit 3, Lesson 6)

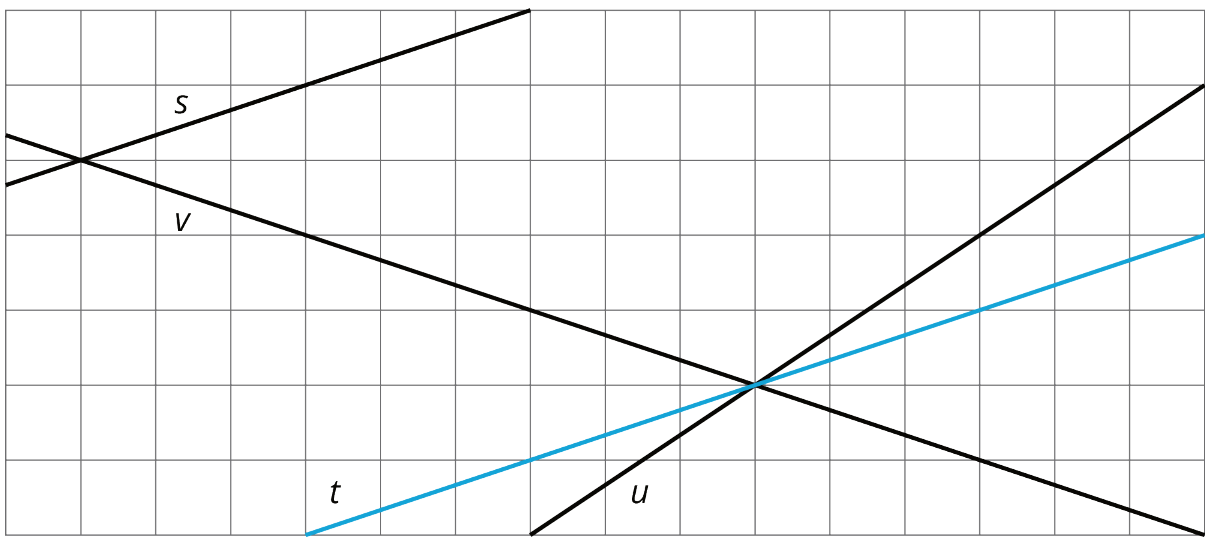
NAME DATE PERIOD

Unit 3, Lesson 9: Slopes Don't Have to be Positive

Let's find out what a negative slope means.

9.1: Which One Doesn't Belong: Odd Line Out

Which line doesn't belong?



9.2: Stand Clear of the Closing Doors, Please

m.openup.org/1/8-3-9-2

Noah put $40 on his fare card. Every time he rides public transportation, $2.50 is subtracted from the amount available on his card.

1. How much money, in dollars, is available on his card after he takes
    a. 0 rides?

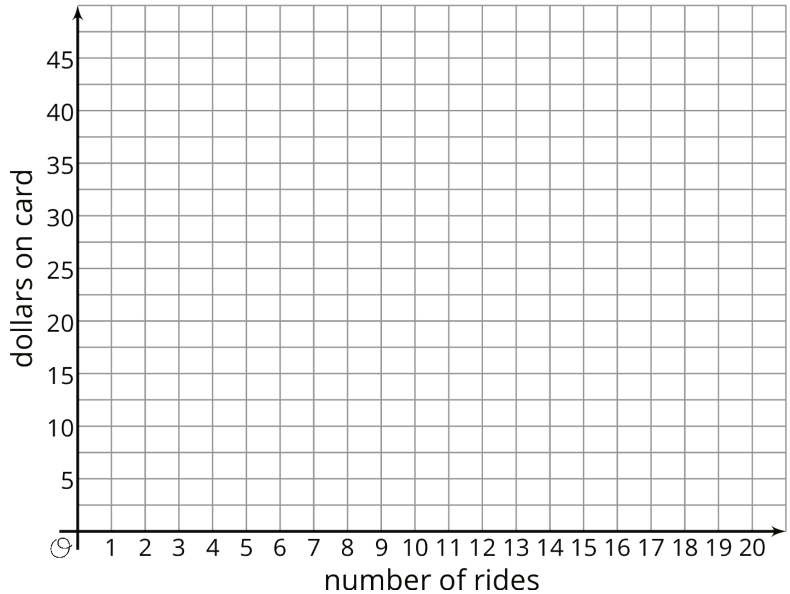
    b. 1 ride?

    c. 2 rides?

    d. x rides?

NAME DATE PERIOD

2. Graph the relationship between amount of money on the card and number of rides.

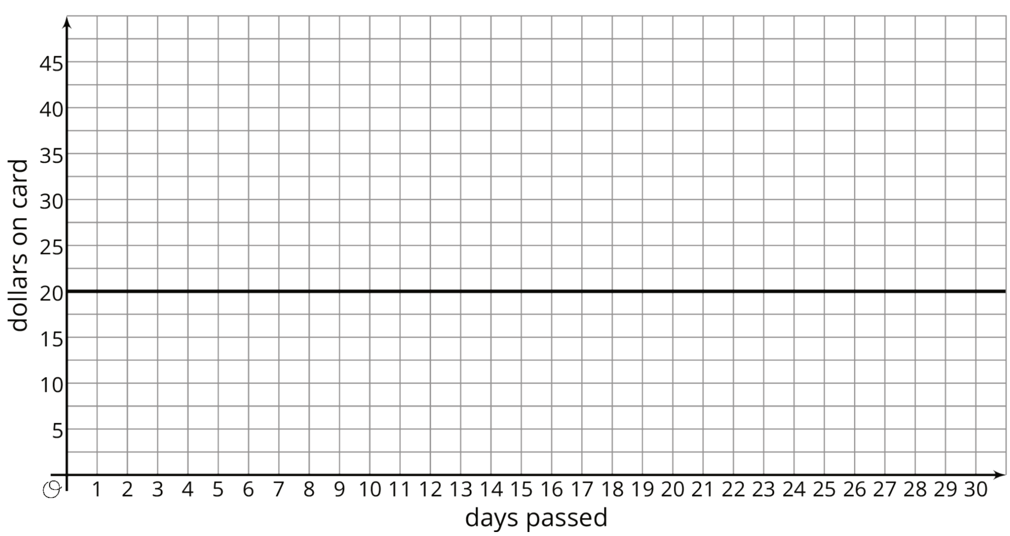


3. How many rides can Noah take before the card runs out of money? Where do you see this number of rides on your graph?

9.3: Travel Habits in July

m.openup.org/1/8-3-9-3

Here is a graph that shows the amount on Han's fare card for every day of last July.



NAME DATE PERIOD

1. Describe what happened with the amount on Han’s fare card in July.

2. Plot and label 3 different points on the line.

3. Write an equation that represents the amount on the card in July, y, after x days.

4. What value makes sense for the slope of the line that represents the amounts on Han’s fare card in July?

Are you ready for more?

Let’s say you have taken out a loan and are paying it back. Which of the following graphs have positive slope and which have negative slope?

1. Amount paid on the vertical axis and time since payments started on the horizontal axis.

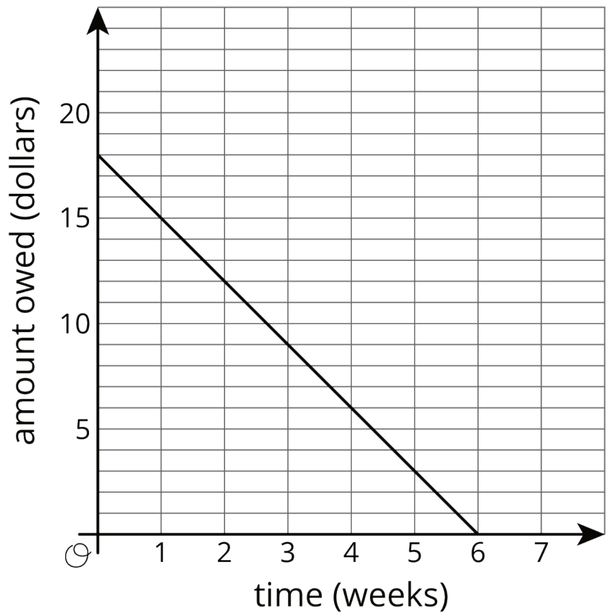
2. Amount owed on the vertical axis and time remaining until the loan is paid off on the horizontal axis.

3. Amount paid on the vertical axis and time remaining until the loan is paid off on the horizontal axis.

NAME DATE PERIOD

9.4: Payback Plan

Elena borrowed some money from her brother. She pays him back by giving him the same amount every week. The graph shows how much she owes after each week.



Answer and explain your reasoning for each question.

1. What is the slope of the line?

2. Explain how you know whether the slope is positive or negative.

3. What does the slope represent in this situation?

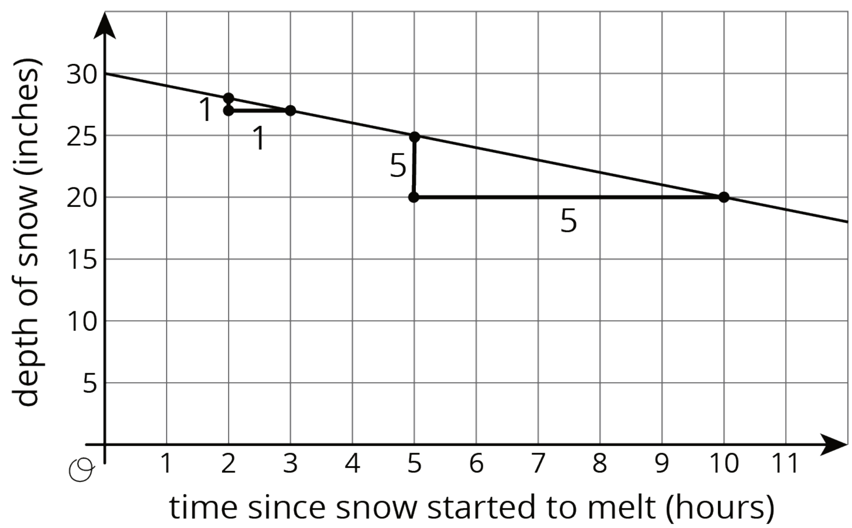
4. How much did Elena borrow?

5. How much time will it take for Elena to pay back all the money she borrowed?

NAME DATE PERIOD

Lesson 9 Summary

At the end of winter in Maine, the snow on the ground was 30 inches deep. Then there was a particularly warm day and the snow melted at the rate of 1 inch per hour. The graph shows the relationship between the time since the snow started to melt and the depth of the snow.



The slope of the graph is -1 since the rate of change is -1 inch per hour. That is, the depth goes *down* 1 inch per hour. The vertical intercept is 30 since the snow was 30 inches deep when the warmth started to melt the snow. The two slope triangles show how the rate of change is constant. It just also happens to be negative in this case since after each hour that passes, there is 1 inch *less* snow.

Graphs with negative slope often describe situations where some quantity is decreasing over time, like the depth of snow on warm days or the amount of money on a fare card being used to take rides on buses.

NAME DATE PERIOD

Slopes can be positive, negative, or even zero! A slope of 0 means there is no change in the y-value even though the x-value may be changing. For example, Elena won a contest where the prize was a special pass that gives her free bus rides for a year. Her fare card had \$5 on it when she won the prize. Here is a graph of the amount of money on her fare card after winning the prize:

The vertical intercept is 5, since the graph starts when she has \$5 on her fare card. The slope of the graph is 0 since she doesn't use her fare card for the next year, meaning the amount on her fare card doesn't change for a year. In fact, all graphs of linear relationships with slopes equal to 0 are horizontal—a rate of change of 0 means that, from one point to the next, the y-values remain the same.

NAME DATE PERIOD

Unit 3, Lesson 9: Slopes Don't Have to be Positive

1. Suppose that during its flight, the elevation e (in feet) of a certain airplane and its time t, in minutes since takeoff, are related by a linear equation. Consider the graph of this equation, with time represented on the horizontal axis and elevation on the vertical axis. For each situation, decide if the slope is positive, zero, or negative.

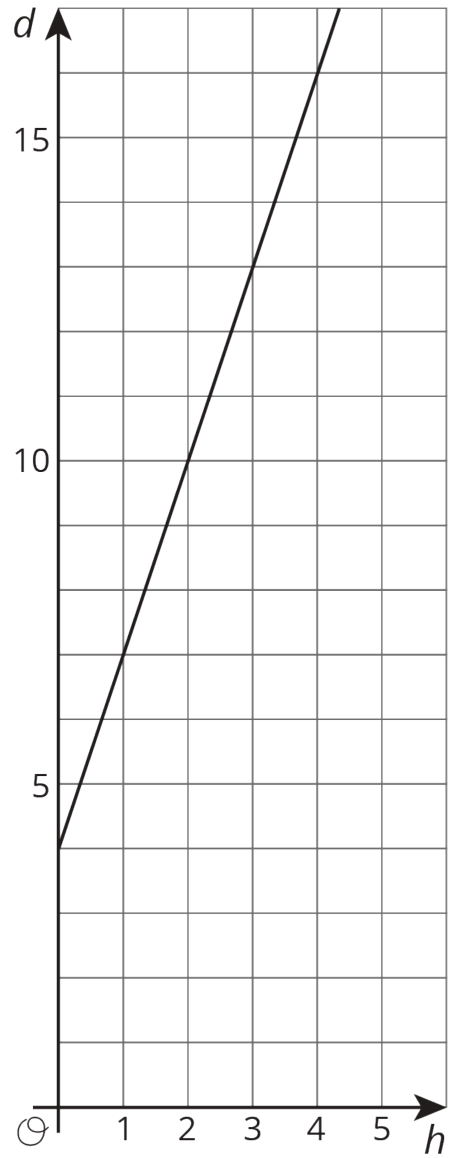
    a. The plane is cruising at an altitude of 37,000 feet above sea level.

    b. The plane is descending at rate of 1000 feet per minute.

    c. The plane is ascending at a rate of 2000 feet per minute.

2. A group of hikers park their car at a trail head and hike into the forest to a campsite. The next morning, they head out on a hike from their campsite walking at a steady rate. The graph shows their distance in miles, d, from the car on the day of their hike after h hours.

NAME DATE PERIOD



a. How far is the campsite from their car? Explain how you know.

b. Write an equation that describes the relationship between d and h.

c. After how many hours will the hikers be 16 miles from their car? Explain or show your reasoning.

(from Unit 3, Lesson 7)

3. Elena's aunt pays her $1 for each call she makes to let people know about her aunt's new business. The table shows how much money Diego receives for washing windows for his neighbors.

NAME DATE PERIOD

number of windows	number of dollars
27	30
45	50
81	90

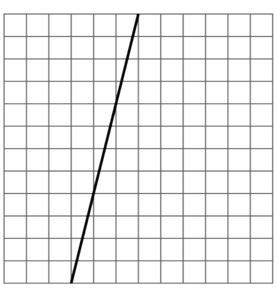
Select **all** the statements about the situation that are true.

A. Elena makes more money for making 10 calls than Diego makes for washing 10 windows.
B. Diego makes more money for washing each window than Elena makes for making each call.
C. Elena makes the same amount of money for 20 calls as Diego makes for 18 windows.
D. Diego needs to wash 35 windows to make as much money as Elena makes for 40 calls.
E. The equation $y = \frac{9}{10}x$, where y is number of dollars and x is number of windows, represents Diego's situation.
F. The equation $y = x$, where y is the number of dollars and x is the number of calls, represents Elena's situation.

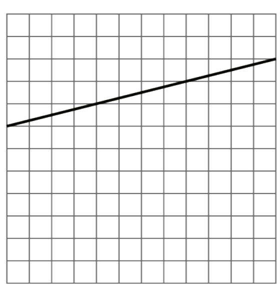
(from Unit 3, Lesson 4)

4. Each square on a grid represents 1 unit on each side. Match the numbers with the slopes of the lines.

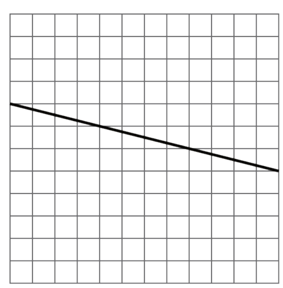
A



B



C



1. $-\frac{1}{4}$
2. $\frac{1}{4}$
3. 4

NAME DATE PERIOD

Unit 3, Lesson 10: Calculating Slope

Let's calculate slope from two points.

10.1: Integer Operations Review

Find values for a and b that make each side have the same value.

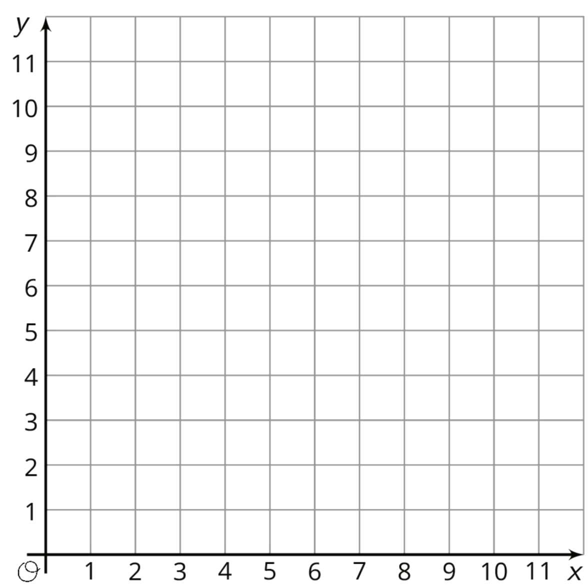
1. $\frac{a}{b} = -2$

2. $\frac{a}{b} = 2$

3. $a - b = -2$

10.2: Toward a More General Slope Formula

1. Plot the points $(1, 11)$ and $(8, 2)$, and use a ruler to draw the line that passes through them.



2. Without calculating, do you expect the slope of the line through $(1, 11)$ and $(8, 2)$ to be positive or negative? How can you tell?

3. Calculate the slope of this line.

NAME DATE PERIOD

Are you ready for more?

Find the value of k so that the line passing through each pair of points has the given slope.

1. $(k, 2)$ and $(11, 14)$, slope = 2
2. $(1, k)$ and $(4, 1)$, slope = -2
3. $(3, 5)$ and $(k, 9)$, slope = $\frac{1}{2}$
4. $(-1, 4)$ and $(-3, k)$, slope = $\frac{-1}{2}$
5. $(\frac{-15}{2}, \frac{3}{16})$ and $(\frac{-13}{22}, k)$, slope = 0

10.3: Making Designs

Your teacher will give you either a design or a blank graph. Do not show your card to your partner.

If your teacher gives you the design:

1. Look at the design silently and think about how you could communicate what your partner should draw. Think about ways that you can describe what a line looks like, such as its slope or points that it goes through.
2. Describe each line, one at a time, and give your partner time to draw them.
3. Once your partner thinks they have drawn all the lines you described, only then should you show them the design.

If your teacher gives you the blank graph:

1. Listen carefully as your partner describes each line, and draw each line based on their description.
2. You are not allowed to ask for more information about a line than what your partner tells you.
3. Do not show your drawing to your partner until you have finished drawing all the lines they describe.

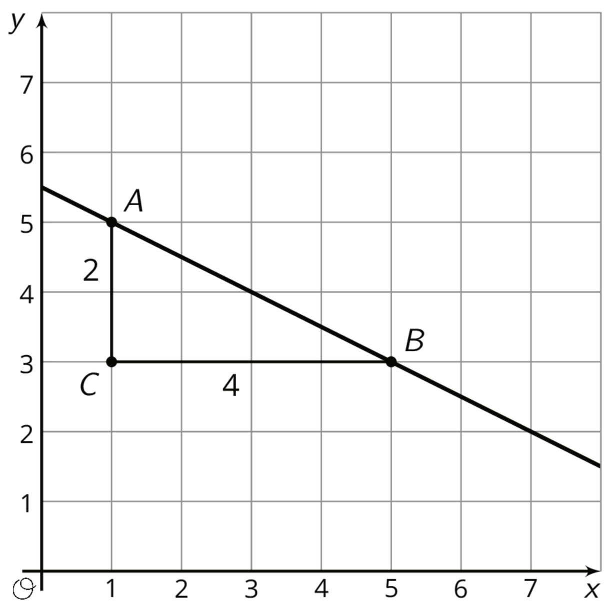
When finished, place the drawing next to the card with the design so that you and your partner can both see them. How is the drawing the same as the design? How is it different? Discuss any miscommunication that might have caused the drawing to look different from the design.

Pause here so your teacher can review your work. When your teacher gives you a new set of cards, switch roles for the second problem.

NAME DATE PERIOD

Lesson 10 Summary

We learned earlier that one way to find the slope of a line is by drawing a slope triangle. For example, using the slope triangle shown here, the slope of the line is $-\frac{2}{4}$, or $-\frac{1}{2}$ (we know the slope is negative because the line is decreasing from left to right).



But slope triangles are only one way to calculate the slope of a line. Let's compute the slope of this line a different way using just the points $A = (1, 5)$ and $B = (5, 3)$. Since we know the slope is the vertical change divided by the horizontal change, we can calculate the change in the y-values and then the change in the x-values. Between points A and B, the y-value change is $3 - 5 = -2$ and the x-value change is $5 - 1 = 4$. This means the slope is $-\frac{2}{4}$, or $-\frac{1}{2}$, which is the same as what we found using the slope triangle.

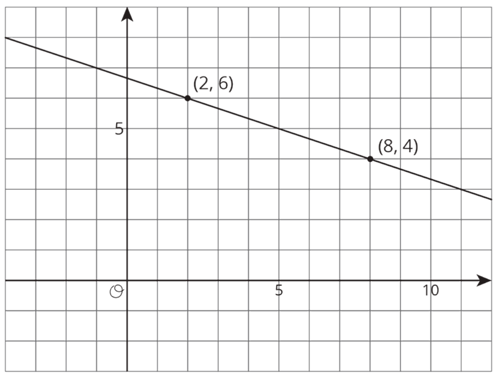
Notice that in each of the calculations, We subtracted the value from point A from the value from point B. If we had done it the other way around, then the y-value change would have been $5 - 3 = 2$ and the x-value change would have been $1 - 5 = -4$, which still gives us a slope of $-\frac{1}{2}$. But what if we were to mix up the orders? If that had happened, we would think the slope of the line is *positive* $\frac{1}{2}$ since we would either have calculated $\frac{-2}{-4}$ or $\frac{2}{4}$. Since we already have a graph of the line and can see it has a negative slope, this is clearly incorrect. It we don't have a graph to check our calculation, we could think about how the point on the left, $(1, 5)$, is higher than the point on the right, $(5, 3)$, meaning the slope of the line must be negative.

NAME DATE PERIOD

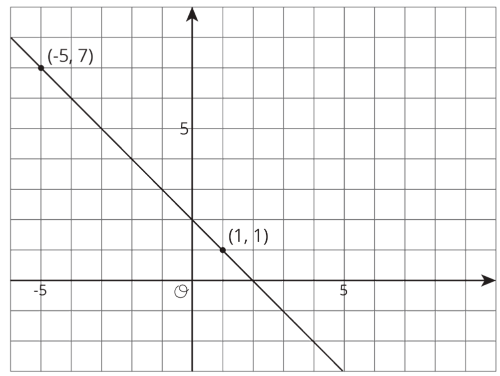
Unit 3, Lesson 10: Calculating Slope

1. For each graph, calculate the slope of the line.

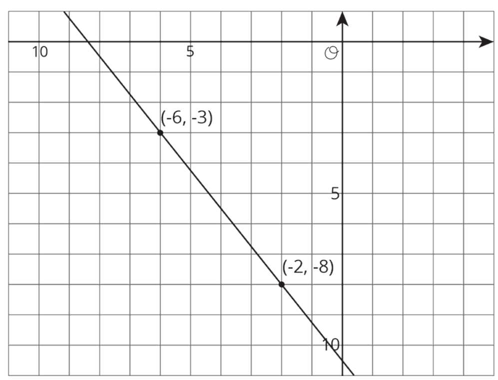
A



B



C



2. Match each pair of points to the slope of the line that joins them.

A. 4

B. -3

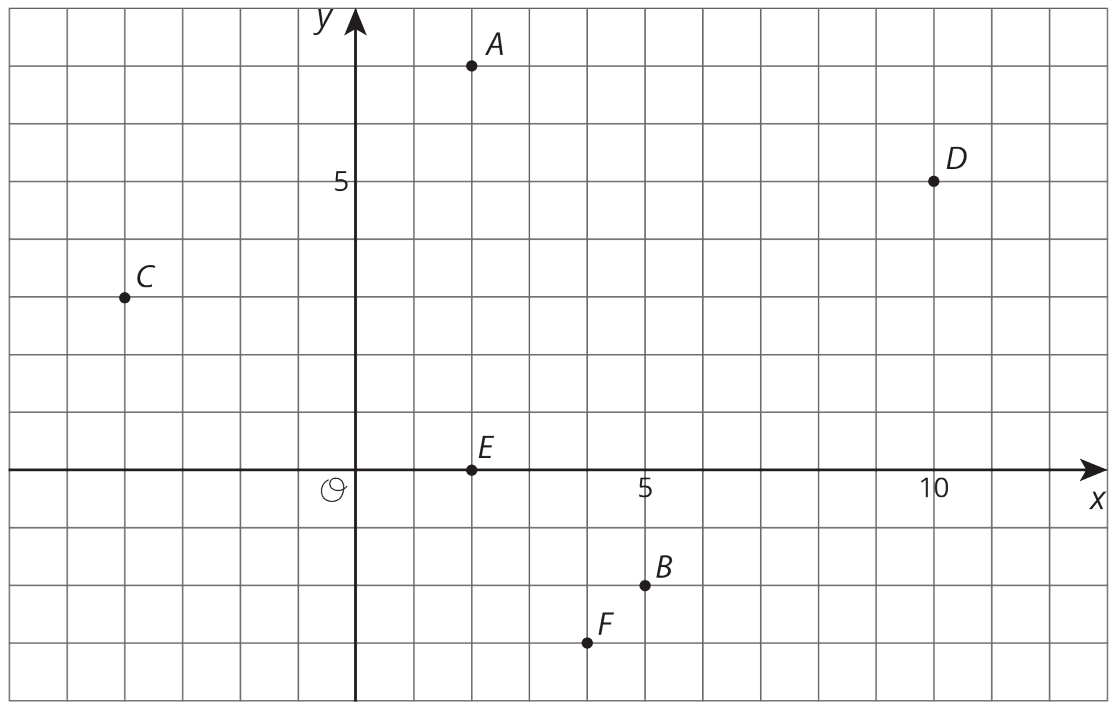
C. $-\frac{5}{2}$

D. $\frac{6}{7}$

1. $(9, 10)$ and $(7, 2)$
2. $(-8, -11)$ and $(-1, -5)$
3. $(5, -6)$ and $(2, 3)$
4. $(6, 3)$ and $(5, -1)$
5. $(4, 7)$ and $(6, 2)$

NAME DATE PERIOD

3. Draw a line with the given slope through the given point. What other point lies on that line?



a. Point A, slope = -3

b. Point A, slope = $\frac{-1}{4}$

c. Point C, slope = $\frac{-1}{2}$

d. Point E, slope = $\frac{-2}{3}$

4. Make a sketch of a linear relationship with a slope of 4 and a negative y-intercept. Show how you know the slope is 4 and write an equation for the line.

(from Unit 3, Lesson 8)

NAME DATE PERIOD

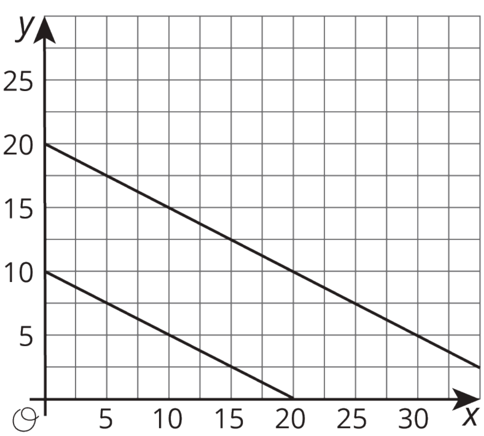
Unit 3, Lesson 11: Equations of All Kinds of Lines

Let's write equations for vertical and horizontal lines.

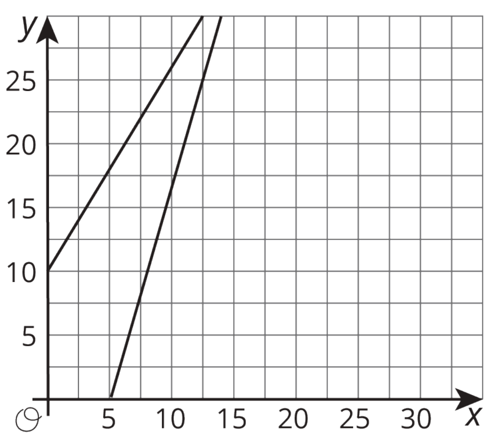
11.1: Which One Doesn't Belong: Pairs of Lines

Which one doesn't belong?

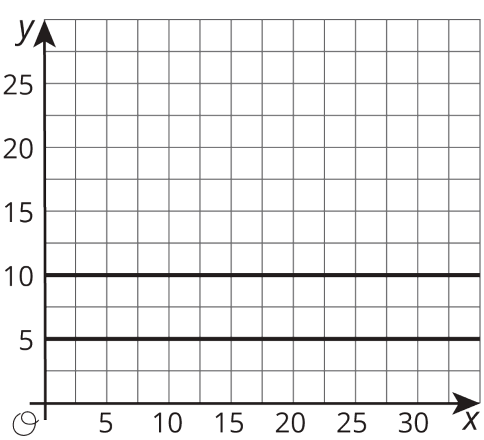
A



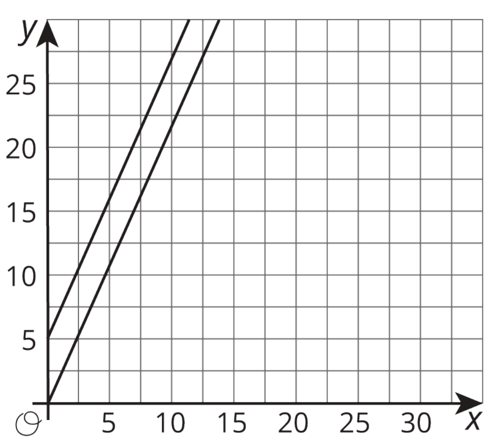
B



C



D



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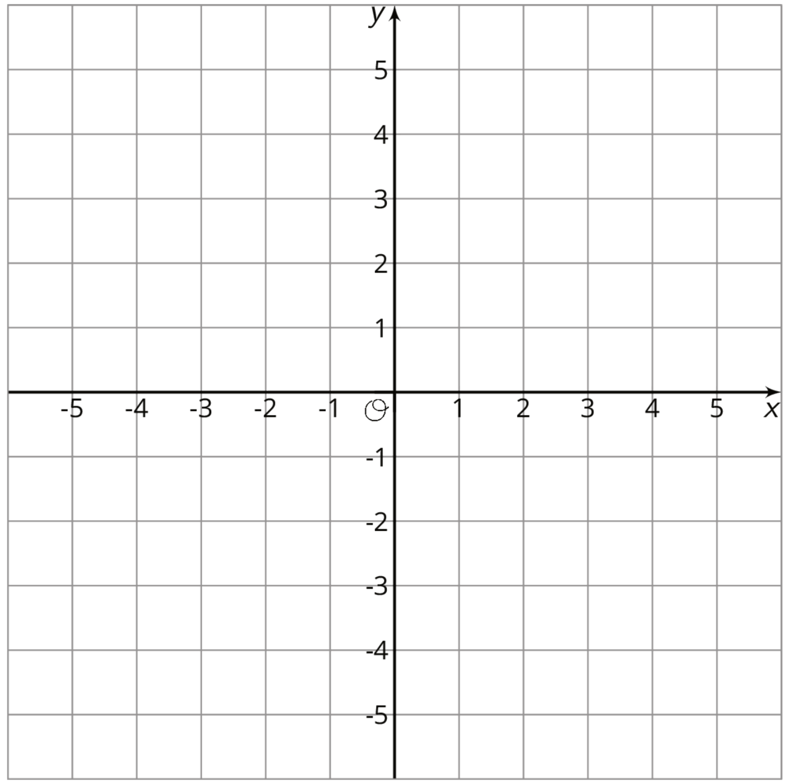
NAME DATE PERIOD

11.2: All the Same



1. Plot at least 10 points whose y-coordinate is -4. What do you notice about them?

2. Which equation makes the most sense to represent all of the points with y-coordinate -4? Explain how you know.

$x = -4$ $\quad$ $y = -4x$ $\quad$ $y = -4$ $\quad$ $x + y = -4$

NAME DATE PERIOD

3. Plot at least 10 points whose x-coordinate is 3. What do you notice about them?

4. Which equation makes the most sense to represent all of the points with x-coordinate 3? Explain how you know.

   $x = 3$ $y = 3x$ $y = 3$ $x + y = 3$

5. Graph the equation $x = -2$.

6. Graph the equation $y = 5$.

Are you ready for more?

1. Draw the rectangle with vertices $(2, 1)$, $(5, 1)$, $(5, 3)$, $(2, 3)$.

2. For each of the four sides of the rectangle, write an equation for a line containing the side.

3. A rectangle has sides on the graphs of $x = -1$, $x = 3$, $y = -1$, $y = 1$. Find the coordinates of each vertex.

NAME DATE PERIOD

11.3: Same Perimeter

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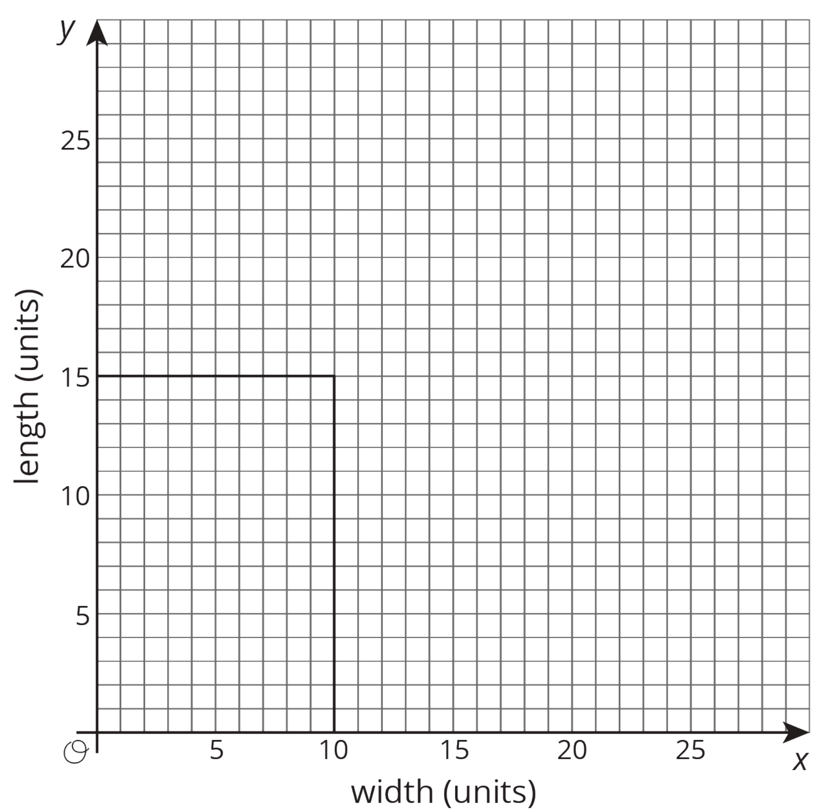
1. There are many possible rectangles whose perimeter is 50 units. Complete the table with lengths, ℓ, and widths, w, of at least 10 such rectangles.

ℓ										
w										

2. The graph shows one rectangle whose perimeter is 50 units, and has its lower left vertex at the origin and two sides on the axes. On the same graph, draw more rectangles with perimeter 50 units using the values from your table. Make sure that each rectangle has a lower left vertex at the origin and two sides on the axes.



3. Each rectangle has a vertex that lies in the first quadrant. These vertices lie on a line. Draw in this line and write an equation for it.

4. What is the the slope of this line? How does the slope describe how the width changes as the length changes (or vice versa)?

NAME DATE PERIOD

Lesson 11 Summary

Horizontal lines in the coordinate plane represent situations where the y value doesn't change at all while the x value changes. For example, the horizontal line that goes through the point $(0, 13)$ can be described in words as "for all points on the line, the y value is always 13." An equation that says the same thing is $y = 13$.

Vertical lines represent situations where the x value doesn't change at all while the y value changes. The equation $x = \text{-}4$ describes a vertical line through the point $(\text{-}4, 0)$.

NAME DATE PERIOD

Unit 3, Lesson 11: Equations of All Kinds of Lines

1. Suppose you wanted to graph the equation $y = -4x - 1$.

   a. Describe the steps you would take to draw the graph.

   b. How would you check that the graph you drew is correct?

2. Draw the following lines and then write an equation for each.

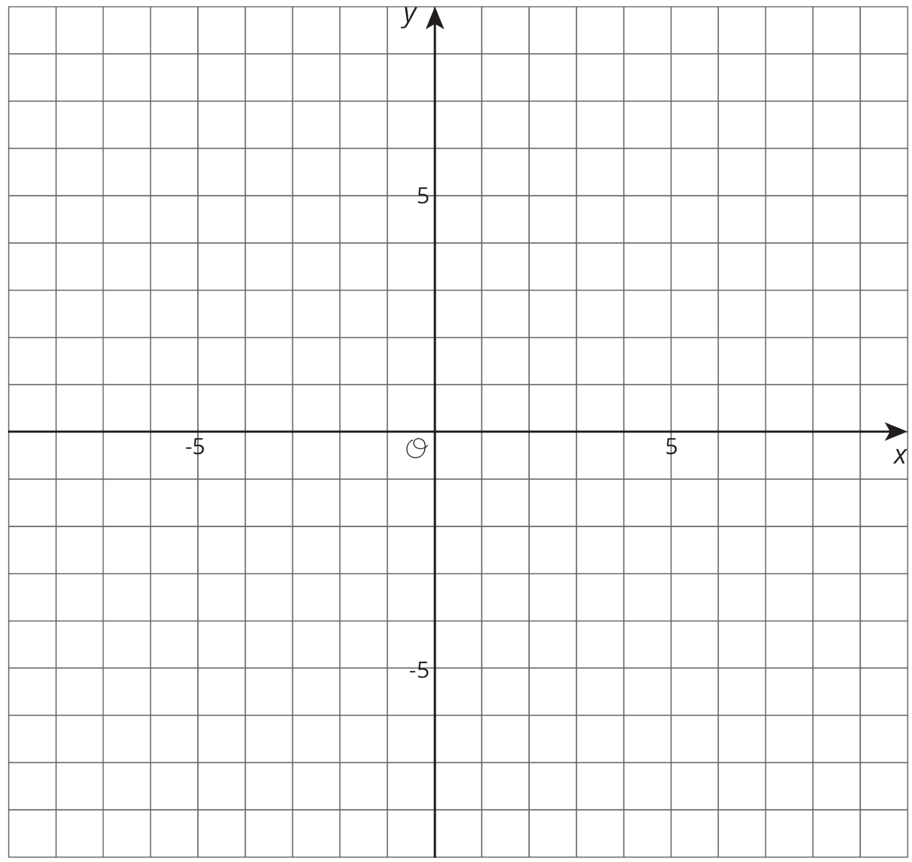
   a. Slope is 0, y-intercept is 5

   b. Slope is 2, y-intercept is -1

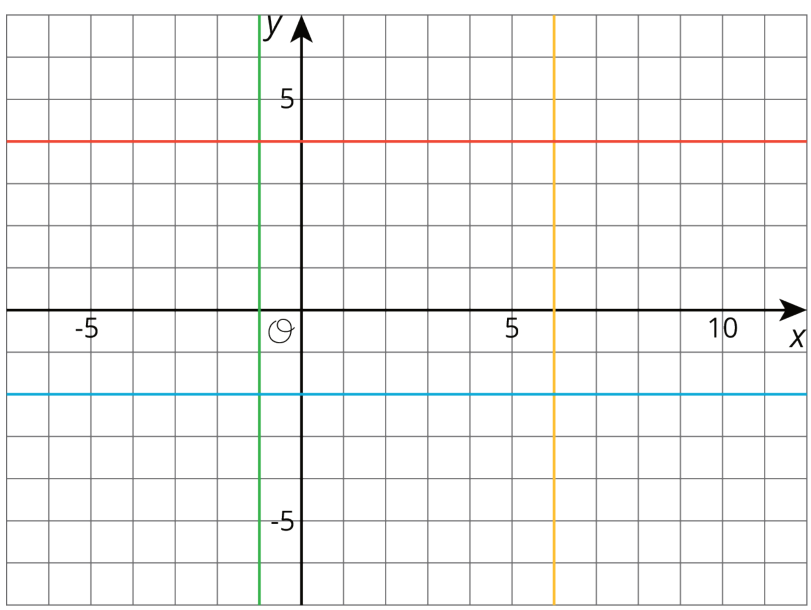
   c. Slope is -2, y-intercept is 1

   d. Slope is $\frac{-1}{2}$, y-intercept is -1

NAME DATE PERIOD



3. Write an equation for each line.



NAME DATE PERIOD

4. A publisher wants to figure out how thick their new book will be. The book has a front cover and a back cover, each of which have a thickness of $\frac{1}{4}$ of an inch. They have a choice of which type of paper to print the book on.

   a. Bond paper has a thickness of $\frac{1}{4}$ inch per one hundred pages. Write an equation for the width of the book, y, if it has x hundred pages, printed on bond paper.

   b. Ledger paper has a thickness of $\frac{2}{5}$ inch per one hundred pages. Write an equation for the width of the book, y, if it has x hundred pages, printed on ledger paper.

   c. If they instead chose front and back covers of thickness $\frac{1}{3}$ of an inch, how would this change the equations in the previous two parts?

   (from Unit 3, Lesson 7)

NAME DATE PERIOD

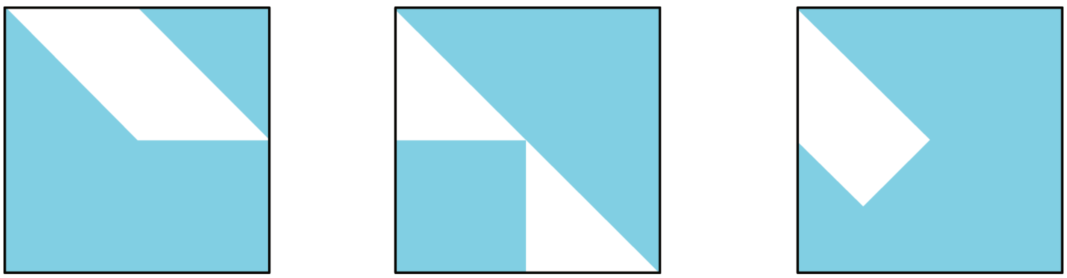
Unit 3, Lesson 12: Solutions to Linear Equations

Let's think about what it means to be a solution to a linear equation with two variables in it.

12.1: Estimate Area

Which figure has the largest shaded region?

A B C



12.2: Apples and Oranges

At the corner produce market, apples cost \$1 each and oranges cost \$2 each.

1. Find the cost of:
    a. 6 apples and 3 oranges

    b. 4 apples and 4 oranges

    c. 5 apples and 4 oranges

    d. 8 apples and 2 oranges

2. Noah has \$10 to spend at the produce market. Can he buy 7 apples and 2 oranges? Explain or show your reasoning.

3. What combinations of apples and oranges can Noah buy if he spends all of his \$10?

NAME DATE PERIOD

4. Use two variables to write an equation that represents \$10-combinations of apples and oranges. Be sure to say what each variable means.

5. What are 3 combinations of apples and oranges that make your equation true? What are three combinations of apples and oranges that make it false?

Are you ready for more?

1. Graph the equation you wrote relating the number of apples and the number of oranges.
2. What is the slope of the graph? What is the meaning of the slope in terms of the context?
3. Suppose Noah has \$20 to spend. Graph the equation describing this situation. What do you notice about the relationship between this graph and the earlier one?

NAME DATE PERIOD

12.3: Solutions and Everything Else

You have two numbers. If you double the first number and add it to the second number, the sum is 10.

1. Let x represent the first number and let y represent the second number. Write an equation showing the relationship between x, y, and 10.

2. Draw and label a set of x- and y-axes. Plot at least five points on this coordinate plane that make the statement and your equation true. What do you notice about the points you have plotted?

3. List ten points that do *not* make the statement true. Using a different color, plot each point in the same coordinate plane. What do you notice about these points compared to your first set of points?

NAME DATE PERIOD

Lesson 12 Summary

Think of all the rectangles whose perimeters are 8 units. If x represents the width and y represents the length, then

$$2x + 2y = 8$$

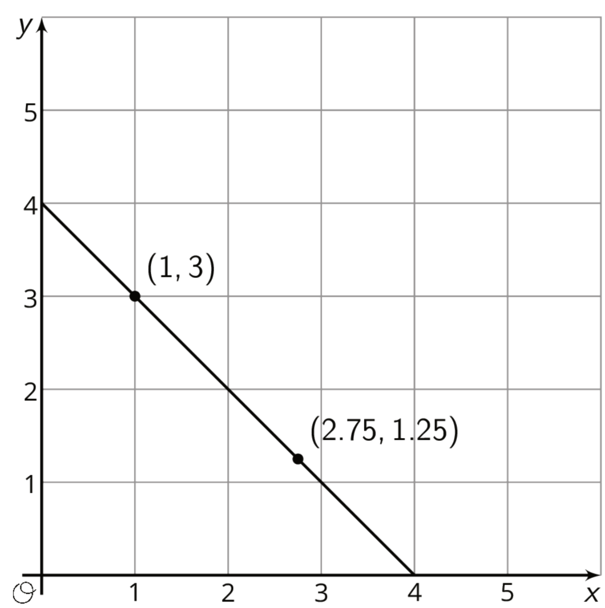
expresses the relationship between the width and length for all such rectangles.

For example, the width and length could be 1 and 3, since $2 \cdot 1 + 2 \cdot 3 = 8$ or the width and length could be 2.75 and 1.25, since $2 \cdot (2.75) + 2 \cdot (1.25) = 8$.

We could find many other possible pairs of width and length, (x, y), that make the equation true—that is, pairs (x, y) that when substituted into the equation make the left side and the right side equal.

A **solution to an equation with two variables** is any pair of values (x, y) that make the equation true.

We can think of the pairs of numbers that are solutions of an equation as points on the coordinate plane. Here is a line created by all the points (x, y) that are solutions to $2x + 2y = 8$. Every point on the line represents a rectangle whose perimeter is 8 units. All points not on the line are not solutions to $2x + 2y = 8$.



Lesson 12 Glossary Terms

- solution to an equation with two variables

NAME DATE PERIOD

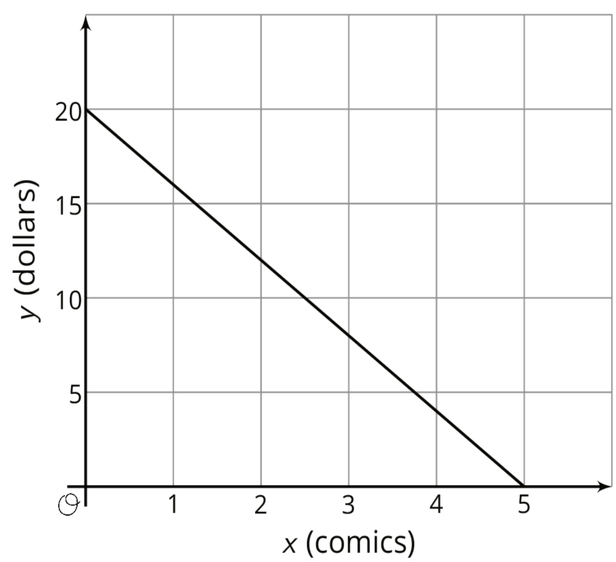
Unit 3, Lesson 12: Solutions to Linear Equations

1. Select **all** of the ordered pairs (x, y) that are solutions to the linear equation $2x + 3y = 6$.

   A. $(0, 2)$
   B. $(0, 6)$
   C. $(2, 3)$
   D. $(3, -2)$
   E. $(3, 0)$
   F. $(6, -2)$

2. The graph shows a linear relationship between x and y.

   x represents the number of comic books Priya buys at the store, all at the same price, and y represents the amount of money (in dollars) Priya has after buying the comic books.



   a. Find and interpret the x- and y-intercepts of this line.

   b. Find and interpret the slope of this line.

   c. Find an equation for this line.

   d. If Priya buys 3 comics, how much money will she have remaining?

3. Match each equation with its three solutions.

NAME DATE PERIOD

A. $y = 1.5x$

B. $2x + 3y = 7$

C. $x - y = 4$

D. $3x = \frac{y}{2}$

E. $y = -x + 1$

1. $(14, 21)$, $(2, 3)$, $(8, 12)$

2. $(-3, -7)$, $(0, -4)$, $(-1, -5)$

3. $\left(\frac{1}{2}, \frac{1}{2}\right)$, $\left(\frac{1}{4}, \frac{3}{4}\right)$, $\left(\frac{1}{8}, \frac{7}{8}\right)$

4. $\left(1, 1\frac{2}{3}\right)$, $(-1, 3)$, $\left(0, 2\frac{1}{3}\right)$

5. $(0.5, 3)$, $(1, 6)$, $(1.2, 7.2)$

4. A container of fuel dispenses fuel at the rate of 5 gallons per second. If y represents the amount of fuel remaining in the container, and x represents the number of seconds that have passed since the fuel started dispensing, then x and y satisfy a linear relationship.

   In the coordinate plane, will the slope of the line representing that relationship have a positive, negative, or zero slope? Explain how you know.

   (from Unit 3, Lesson 10)

5. A sandwich store charges a delivery fee to bring lunch to an office building. One office pays $33 for 4 turkey sandwiches. Another office pays $61 for 8 turkey sandwiches. How much does each turkey sandwich add to the cost of the delivery? Explain how you know.

   (from Unit 3, Lesson 5)

NAME DATE PERIOD

Unit 3, Lesson 13: More Solutions to Linear Equations

Let’s find solutions to more linear equations.

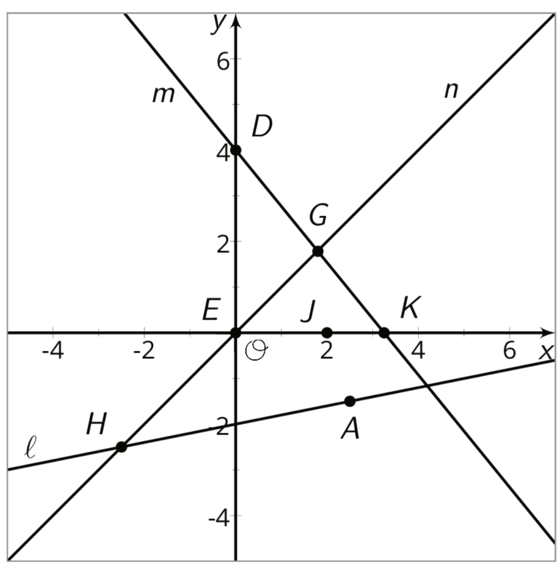
13.1: Coordinate Pairs

For each equation choose a value for x and then solve to find the corresponding y value that makes that equation true.

1. $6x = 7y$
2. $5x + 3y = 9$
3. $y + 5 - \frac{1}{3}x = 7$

13.2: True or False: Solutions in the Coordinate Plane

Here are graphs representing three linear relationships. These relationships could also be represented with equations.



For each statement below, decide if it is true or false. Explain your reasoning.

1. $(4, 0)$ is a solution of the equation for line m.
2. The coordinates of the point G make both the equation for line m and the equation for line n true.

NAME DATE PERIOD

3. $x = 0$ is a solution of the equation for line n.

4. $(2, 0)$ makes both the equation for line m and the equation for line n true.

5. There is no solution for the equation for line ℓ that has $y = 0$.

6. The coordinates of point H are solutions to the equation for line ℓ.

7. There are exactly two solutions of the equation for line ℓ.

8. There is a point whose coordinates make the equations of all three lines true.

After you finish discussing the eight statements, find another group and check your answers against theirs. Discuss any disagreements.

13.3: I'll Take an X, Please

One partner has 6 cards labeled A through F and one partner has 6 cards labeled a through f. In each pair of cards (for example, Cards A and a), there is an equation on one card and a coordinate pair, (x, y), that makes the equation true on the other card.

1. The partner with the equation asks the partner with a solution for either the x-value or the y-value and explains why they chose the one they did.

2. The partner with the equation uses this value to find the other value, explaining each step as they go.

3. The partner with the coordinate pair then tells the partner with the equation if they are right or wrong. If they are wrong, both partners should look through the steps to find and correct any errors. If they are right, both partners move onto the next set of cards.

4. Keep playing until you have finished Cards A through F.

Are you ready for more?

Consider the equation $ax + by = c$, where a, b, and c are positive numbers.

1. Find the coordinates of the x- and y-intercepts of the graph of the equation.

NAME DATE PERIOD

2. Find the slope of the graph.

Lesson 13 Summary

Let's think about the linear equation $2x - 4y = 12$. If we know $(0, \text{-}3)$ is a solution to the equation, then we also know $(0, \text{-}3)$ is a point on the graph of the equation. Since this point is on the y-axis, we also know that it is the vertical intercept of the graph. But what about the coordinate of the horizontal intercept, when $y = 0$? Well, we can use the equation to figure it out.

$$\begin{aligned} 2x - 4y &= 12 \\ 2x - 4(0) &= 12 \\ 2x &= 12 \\ x &= 6 \end{aligned}$$

Since $x = 6$ when $y = 0$, we know the point $(6, 0)$ is on the graph of the line. No matter the form a linear equation comes in, we can always find solutions to the equation by starting with one value and then solving for the other value.

NAME DATE PERIOD

Unit 3, Lesson 13: More Solutions to Linear Equations

1. For each equation, find y when $x = -3$. Then find x when $y = 2$

   a. $y = 6x + 8$

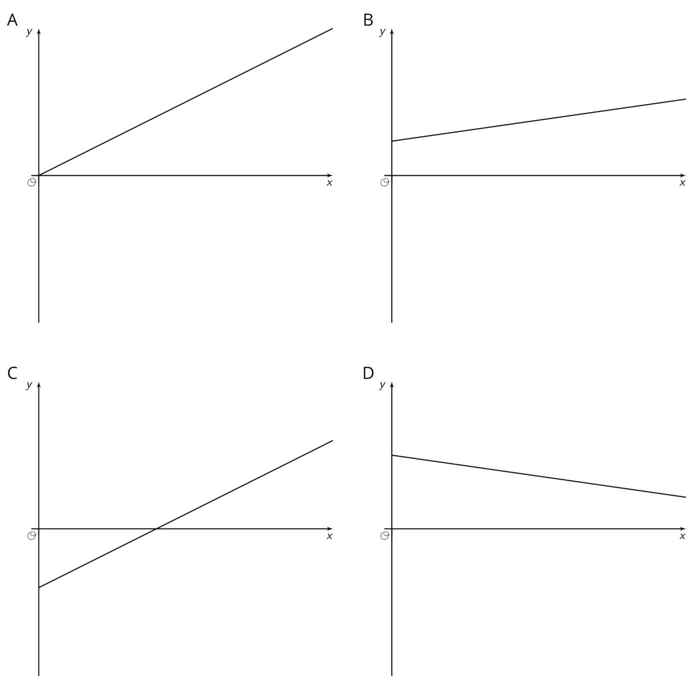
   b. $y = \frac{2}{3}x$

   c. $y = -x + 5$

   d. $y = \frac{3}{4}x - 2\frac{1}{2}$

   e. $y = 1.5x + 11$

2. Match each graph of a linear relationship to a situation that most reasonably reflects its context.



   1. y is the weight of a kitten x days after birth.

   2. y is the distance left to go in a car ride after x hours of driving at a constant rate toward its destination.

   3. y is the temperature, in degrees C, of a gas being warmed in a laboratory experiment.

   4. y is the amount of calories consumed eating x crackers.

   (from Unit 3, Lesson 9)

3. True or false: The points $(6, 13)$, $(21, 33)$, and $(99, 137)$ all lie on the same line. The equation of the line is $y = \frac{4}{3}x + 5$. Explain or show your reasoning.

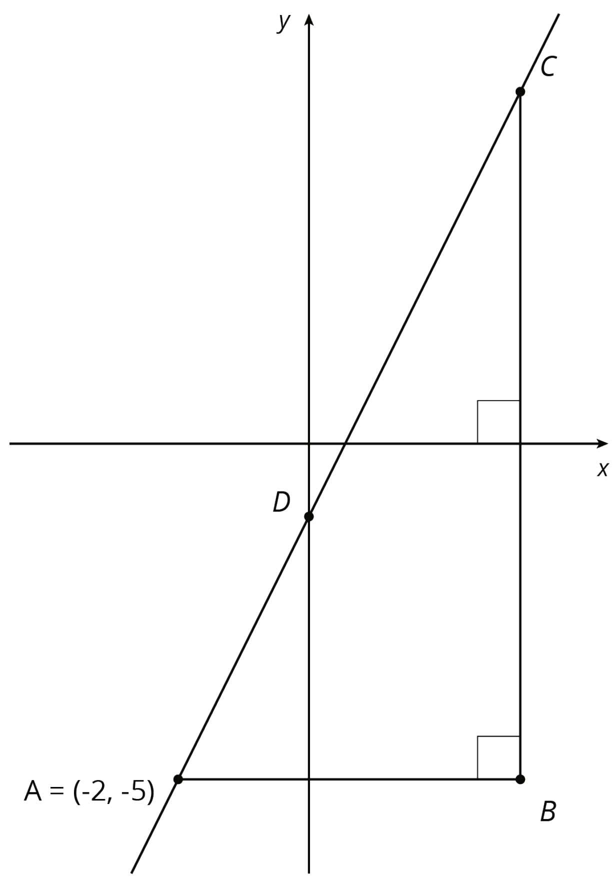
NAME DATE PERIOD

4. Here is a linear equation: $y = \frac{1}{4}x + \frac{5}{4}$

   a. Are $(1, 1.5)$ and $(12, 4)$ solutions to the equation? Explain or show your reasoning.

   b. Find the x-intercept of the graph of the equation. Explain or show your reasoning.

5. Find the coordinates of B, C, and D given that AB = 5 and BC = 10.



(from Unit 2, Lesson 11)

NAME DATE PERIOD

Unit 3, Lesson 14: Using Linear Relations to Solve Problems

Let’s write equations for real-world situations and think about their solutions.

14.1: Buying Fruit

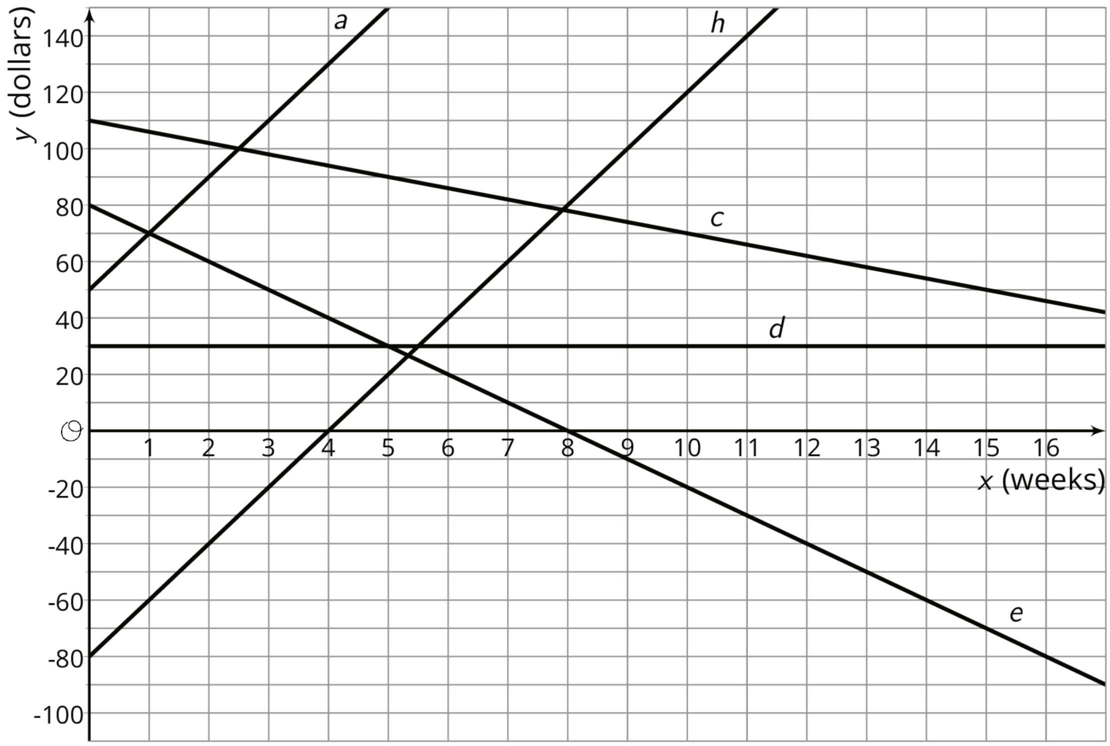
For each relationship described, write an equation to represent the relationship.

1. Grapes cost \$2.39 per pound. Bananas cost \$0.59 per pound. You have \$15 to spend on g pounds of grapes and b pounds of bananas.

2. A savings account has \$50 in it at the start of the year and \$20 is deposited each week. After x weeks, there are y dollars in the account.

NAME DATE PERIOD

14.2: Five Savings Accounts



Each line represents one person's weekly savings account balance from the start of the year.

1. Choose one line and write a description of what happens to that person's account over the first 17 weeks of the year. Do not tell your group which line you chose.

2. Share your story with your group and see if anyone can guess your line.

3. Write an equation for each line on the graph. What do the slope, m, and vertical intercept, b, in each equation mean in the situation?

NAME DATE PERIOD

4. For which equation is $(1, 70)$ a solution? Interpret this solution in terms of your story.

5. Predict the balance in each account after 20 weeks.

14.3: Fabulous Fish

The Fabulous Fish Market orders tilapia, which costs \$3 per pound, and salmon, which costs \$5 per pound. The market budgets \$210 to spend on this order each day.

1. What are five different combinations of salmon and tilapia that the market can order?

2. Define variables and write an equation representing the relationship between the amount of each fish bought and how much the market spends.

NAME DATE PERIOD

3. Sketch a graph of the relationship. Label your axes.

4. On your graph, plot and label the combinations A—F.

	A	B	C	D	E	F
pounds of tilapia	5	19	27	25	65	55
pounds of salmon	36	30.6	25	27	6	4

a. Which of these combinations can the market order? Explain or show your reasoning.

5. List two ways you can tell if a pair of numbers is a solution to an equation.

NAME DATE PERIOD

Unit 3, Lesson 14: Using Linear Relations to Solve Problems

1. The owner of a new restaurant is ordering tables and chairs. He wants to have only tables for 2 and tables for 4. The total number of people that can be seated in the restaurant is 120.

   a. Describe some possible combinations of 2-seat tables and 4-seat tables that will seat 120 customers. Explain how you found them.

   b. Write an equation to represent the situation. What do the variables represent?

   c. Create a graph to represent the situation.

   d. What does the slope tell us about the situation?

   e. Interpret the x and y intercepts in the situation.

2. Triangle A is an isosceles triangle with two angles of measure x degrees and one angle of measure y degrees.

   a. Find three combinations of x and y that make this sentence true.

NAME DATE PERIOD

b. Write an equation relating x and y.

c. If you were to sketch the graph of this linear equation, what would its slope be? How can you interpret the slope in the context of the triangle?

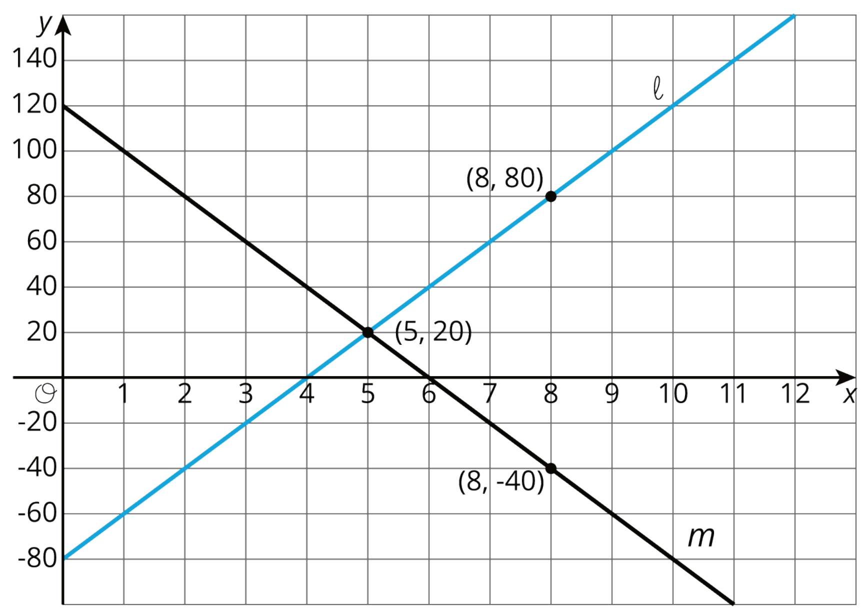
(from Unit 3, Lesson 13)

3. Select **all** the equations for which $(-6, -1)$ is a solution.

A. $y = 4x + 23$
B. $3x = \frac{1}{2}y$
C. $2x - 13y = 1$
D. $3y = \frac{1}{2}x$
E. $2x + 6y = -6$

(from Unit 3, Lesson 12)

4. Consider the following graphs of linear equations. Decide which line has a positive slope, and which has a negative slope. Then calculate each line's exact slope.



(from Unit 3, Lesson 10)