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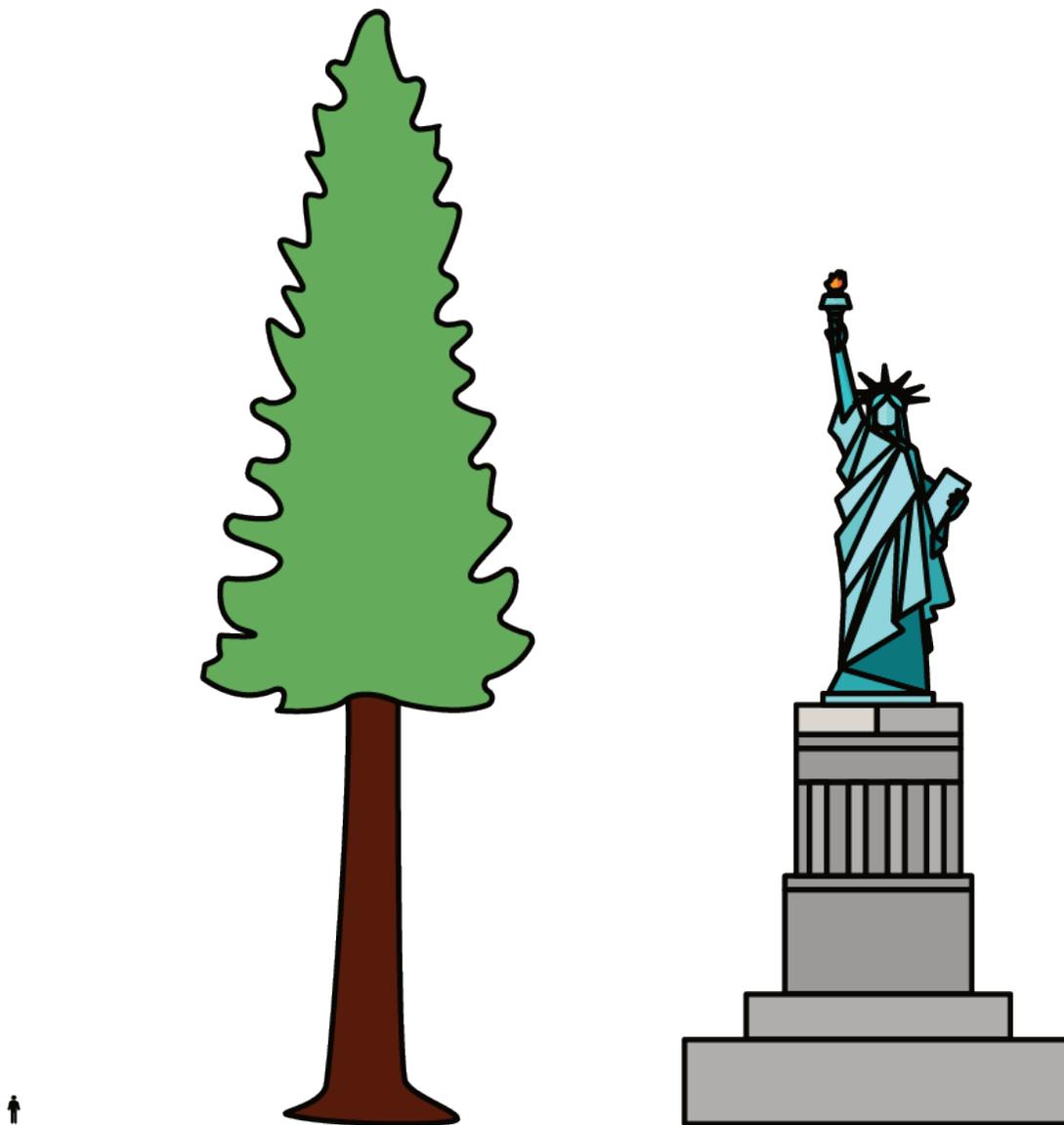
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Unit 3, Lesson 1: The Burj Khalifa

Let's investigate the Burj Khalifa building.

1.1: Estimating Height

Use the picture to estimate the height of Hyperion, the tallest known tree.



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1.2: Window Washing



A window-washing crew can finish 15 windows in 18 minutes.

If this crew was assigned to wash all the windows on the outside of the Burj Khalifa, how long will the crew be washing at this rate?

1.3: Climbing the Burj Khalifa

In 2011, a professional climber scaled the outside of the Burj Khalifa, making it all the way to 828 meters (the highest point on which a person can stand) in 6 hours.

Assuming they climbed at the same rate the whole way:

1. How far did they climb in the first 2 hours?

2. How far did they climb in 5 hours?

3. How far did they climb in the final 15 minutes?

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Are you ready for more?

Have you ever seen videos of astronauts on the moon jumping really high?

An object on the moon weighs less than it does on Earth because the moon has much less mass than Earth.

- A woman who weighs 160 pounds on Earth weighs 30 pounds on the moon. If a boy weighs 60 pounds on Earth, how much does he weigh on the moon?

- Every 10 pounds on Earth are the equivalent to 4 pounds on Mars. If the same woman travels to Mars, how much would she weigh there?

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Lesson 1 Summary

There are many real-world situations in which something keeps happening at the same rate. For example:

- a bus stop that is serviced by 4 buses per hour
- a washing machine that takes 45 minutes per load of laundry
- a school cafeteria that serves 15 students per minute

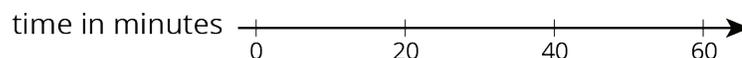
In situations like these, we can use equivalent ratios to predict how long it will take for something to happen some number of times, or how many times it will happen in a particular length of time.

- How long will it take the school cafeteria to serve 600 students?

The table shows that it will take the cafeteria 40 minutes to serve 600 students.

number of students	time in minutes
15	1
60	4
600	40

- How many students can the cafeteria serve in 1 hour?



The double number line shows that the cafeteria can serve 900 students in 1 hour.

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Unit 3, Lesson 1: The Burj Khalifa

1. An elevator travels 310 feet in 10 seconds. At that speed, how far can this elevator travel in 12 seconds? Explain your reasoning.
2. Han earns \$33.00 for babysitting 4 hours. At this rate, how much will he earn if he babysits for 7 hours? Explain your reasoning.
3. The cost of 5 cans of dog food is \$4.35. At this price, how much do 11 cans of dog food cost? Explain your reasoning.
4. A restaurant has 26 tables in its dining room. It takes the waitstaff 10 minutes to clear and set 4 tables. At this rate, how long will it take the waitstaff to clear and set all the tables in the dining room? Explain or show your reasoning.
5. A sandwich shop serves 4 ounces of meat and 3 ounces of cheese on each sandwich. After making sandwiches for an hour, the shop owner has used 91 combined ounces of meat and cheese.
 - a. How many combined ounces of meat and cheese are used on each sandwich?

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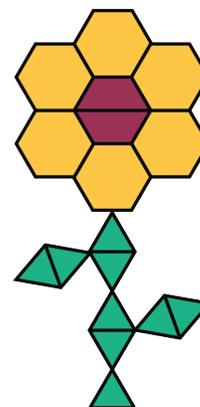
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- b. How many sandwiches were made in the hour?
- c. How many ounces of meat were used?
- d. How many ounces of cheese were used?

(from Unit 2, Lesson 16)

6. Here is a flower made up of yellow hexagons, red trapezoids, and green triangles.

- a. How many copies of this flower pattern could you build if you had 30 yellow hexagons, 50 red trapezoids, and 60 green triangles?
- b. Of which shape would you have the most left over?



(from Unit 2, Lesson 14)

7. Match each quantity in the first list with an appropriate unit of measurement from the second list.

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- | | |
|--------------------------------------|------------------------------|
| A. the perimeter of a baseball field | 1. centimeters (cm) |
| B. the area of a bed sheet | 2. cubic feet (cu ft) |
| C. the volume of a refrigerator | 3. cubic kilometers (cu km) |
| D. the surface area of a tissue box | 4. meters (m) |
| E. the length of a spaghetti noodle | 5. square feet (sq ft) |
| F. the volume of a large lake | 6. square inches (sq in) |
| G. the surface area of the the moon | 7. square kilometers (sq km) |

(from Unit 1, Lesson 16)

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Unit 3, Lesson 2: Anchoring Units of Measurement

Let's see how big different things are.

2.1: Estimating Volume

Estimate the volume of the tiny salt shaker.



2.2: Cutting String

Your teacher will assign you one of the following lengths: 1 centimeter, 1 foot, 1 inch, 1 meter, or 1 yard.

Estimate and cut a piece of string as close to your assigned length as you can without using a measurement tool.

2.3: Card Sort: Measurements

Your teacher will give you some cards with the names of different units of measurement and other cards with pictures of objects.

1. Sort the units of measurement into groups based on the attribute they measure.
Pause here so your teacher can review your groups.
2. Match each picture card that has "L" in the top right corner with the closest unit to the length of the object.
3. Match each picture card that has "V" in the top right corner with the closest unit to the volume of the object.
4. Match each picture card that has "WM" in the top right corner with the closest unit to the weight or mass of the object.

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Your teacher will assign you a new group to discuss how you matched the objects. If you disagree, work to reach an agreement.

Lesson 2 Summary

We can use everyday objects to estimate standard units of measurement.

For units of length:

- 1 millimeter is about the thickness of a dime.
- 1 centimeter is about the width of a pinky finger.
- 1 inch is about the length from the tip of your thumb to the first knuckle.
- 1 foot is the length of a football.
- 1 yard is about the length of a baseball bat.
- 1 meter is about the length of a baseball bat and ball.
- 1 kilometer is about the distance someone walks in ten minutes.
- 1 mile is about the distance someone runs in ten minutes.

For units of volume:

- 1 milliliter is about the volume of a raindrop.
- 1 cup is about the volume of a school milk carton.
- 1 quart is about the volume of a large sports drink bottle.
- 1 liter is about the volume of a reusable water bottle.
- 1 gallon is about the volume of a large milk jug.

For units of weight and mass:

- 1 gram is about the mass of a raisin.
- 1 ounce is about the weight of a slice of bread.
- 1 pound is about the weight of a loaf of bread.
- 1 kilogram is about the mass of a textbook.
- 1 ton is about the weight of a small car.

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Unit 3, Lesson 2: Anchoring Units of Measurement

1. Select the unit from the list that you would use to measure each object.

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A. The length of a pencil		1. centimeters
B. The weight or mass of a pencil		2. cups
C. The volume of a pencil		3. feet
D. The weight or mass of a hippopotamus		4. gallons
E. The length of a hippopotamus		5. grams
F. The length of a fingernail clipping		6. inches
G. The weight or mass of a fingernail clipping		7. kilograms
H. The volume of a sink		8. kilometers
I. The volume of a bowl		9. liters
J. The length of a chalkboard or whiteboard		10. meters
K. The weight or mass of a chalkboard or whiteboard		11. miles
L. The length of the border between the United States and Canada		12. milliliters
		13. millimeters
		14. ounces
		15. pounds
		16. quarts
		17. tons
		18. yards

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2. When this pet hamster is placed on a digital scale, the scale reads 1.5.



What could be the units?

3. Circle the larger unit of measure. Then, determine if the unit measures distance, volume, or weight (mass).

a. meter or kilometer

c. cup or quart

e. liter or milliliter

b. yard or foot

d. pound or ounce

f. gram or kilogram

4. Elena mixes 5 cups of apple juice with 2 cups of sparkling water to make sparkling apple juice. For a party, she wants to make 35 cups of sparkling apple juice. How much of each ingredient should Elena use? Explain or show your reasoning.

(from Unit 2, Lesson 15)

5. Lin bought 3 hats for \$22.50. At this rate, how many hats could she buy with \$60.00? If you get stuck, try using the table.

NAME _____

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number of hats	price in dollars

(from Unit 2, Lesson 12)

6. Light travels about 180 million kilometers in 10 minutes. How far does it travel in 1 minute? How far does it travel in 1 second? Show your reasoning.

(from Unit 2, Lesson 9)

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Unit 3, Lesson 3: Measuring with Different-Sized Units

Let's measure things.

3.1: Width of a Paper

Your teacher will show you two rods. Does it take more green rods or blue rods lined up end to end to measure the width of a piece of printer paper?

3.2: Measurement Stations

m.openup.org/1/6-3-3-2



Station 1

- Each large cube is 1 cubic inch. Count how many cubic inches completely pack the box without gaps.
- Each small cube is 1 cubic centimeter. Each rod is composed of 10 cubic centimeters. Count how many cubic centimeters completely fill the box.

	cubic inches	cubic centimeters
volume of the box		

Station 2

Your teacher showed you a length.

- Use the meter stick to measure the length to the nearest meter.
- Use a ruler to measure the length to the nearest foot.

	meters	feet
length of _____		

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Station 3

If not using real water, open <https://vimeo.com/184901230>.

- Count how many times you can fill the quart bottle from the gallon jug.
- Count how many times you can fill the liter bottle from the gallon jug.

	quarts	liters
1 gallon of water		

Station 4

If not using a real scale, open <http://ggbm.at/eQQVYB7D>.

- Select 2–3 different objects to measure on the scale.
- Record the weights in ounces, pounds, grams, and kilograms.

	ounces	pounds	grams	kilograms
object 1				
object 2				
object 3				

Station 5

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- Count how many level teaspoons of salt fill the graduated cylinder to 20 milliliters, 40 milliliters, and 50 milliliters.
- Pour the salt back into the original container.

	milliliters	teaspoons
small amount of salt	20	
medium amount of salt	40	
large amount of salt	50	

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After you finish all five stations, answer these questions with your group.

1.
 - a. Which is larger, a cubic inch or a cubic centimeter?
 - b. Did more cubic inches or cubic centimeters fit in the cardboard box? Why?

2. Did it take more feet or meters to measure the indicated length? Why?

3. Which is larger, a quart or a liter? Explain your reasoning.

4. Use the data from Station 4 to put the units of weight and mass in order from smallest to largest. Explain your reasoning.

5.
 - a. About how many teaspoons of salt would it take to fill the graduated cylinder to 100 milliliters?
 - b. If you poured 15 teaspoons of salt into an empty graduated cylinder, about how many milliliters would it fill?
 - c. How many milliliters per teaspoon are there?
 - d. How many teaspoons per milliliter are there?

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Are you ready for more?

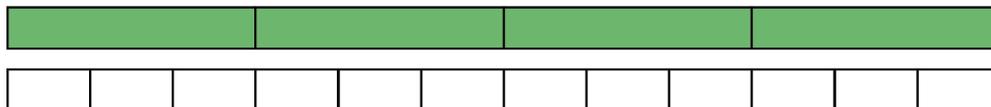
People in the medical field use metric measurements when working with medicine. For example, a doctor might prescribe medication in 10 mg tablets.

Brainstorm a list of reasons why healthcare workers would do this. Organize your thinking so it can be followed by others.

Lesson 3 Summary

The size of the unit we use to measure something affects the measurement.

If we measure the same quantity with different units, it will take more of the smaller unit and fewer of the larger unit to express the measurement. For example, a room that measures 4 yards in length will measure 12 feet.



There are 3 feet in a yard, so one foot is $\frac{1}{3}$ of a yard.

- It takes 3 times as many feet to measure the same length as it does with yards.
- It takes $\frac{1}{3}$ as many yards to measure the same length as it does with feet.

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Unit 3, Lesson 3: Measuring with Different-Sized Units

1. Decide if each is a measurement of length, area, volume, or weight (or mass).

- How many centimeters across a handprint
- How many square inches of paper needed to wrap a box
- How many gallons of water in a fish tank
- How many pounds in a bag of potatoes
- How many feet across a swimming pool
- How many ounces in a bag of grapes
- How many liters in a punch bowl
- How many square feet of grass in a lawn

(from Unit 3, Lesson 2)

2. Clare says, "This classroom is 11 meters long. A meter is longer than a yard, so if I measure the length of this classroom in yards, I will get less than 11 yards." Do you agree with Clare? Explain your reasoning.

3. Tyler's height is 57 inches. What could be his height in centimeters? Explain your reasoning.

- 22.4
- 57
- 144.8
- 3,551

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4. A large soup pot holds 20 quarts. What could be its volume in liters?

- A. 7.57
- B. 19
- C. 21
- D. 75.7

5. Clare wants to mail a package that weighs $4\frac{1}{2}$ pounds. What could this weight be in kilograms?

- A. 2.04
- B. 4.5
- C. 9.92
- D. 4,500

6. Noah bought 15 baseball cards for \$9.00. Assuming each baseball card costs the same amount, answer the following questions.

- a. At this rate, how much will 30 baseball cards cost? Explain your reasoning.
- b. At this rate, how much will 12 baseball cards cost? Explain your reasoning.
- c. Do you think this information would be better represented using a table or a double number line? Explain your reasoning.

(from Unit 2, Lesson 13)

7. Jada traveled 135 miles in 3 hours. Andre traveled 228 miles in 6 hours. Both Jada and Andre traveled at a constant speed.

- a. How far did Jada travel in 1 hour?
- b. How far did Andre travel in 1 hour?
- c. Who traveled faster? Explain or show your reasoning.

(from Unit 2, Lesson 9)

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Unit 3, Lesson 4: Converting Units

Let's convert measurements to different units.

4.1: Number Talk: Fractions of a Number

Find the values mentally.

$$\frac{1}{4} \text{ of } 32$$

$$\frac{3}{4} \text{ of } 32$$

$$\frac{3}{8} \text{ of } 32$$

$$\frac{3}{8} \text{ of } 64$$

4.2: Road Trip

Elena and her mom are on a road trip outside the United States. Elena sees this road sign.

Elena's mom is driving 75 miles per hour when she gets pulled over for speeding.



1. The police officer explains that 8 kilometers is approximately 5 miles.
 - a. How many kilometers are in 1 mile?
 - b. How many miles are in 1 kilometer?
2. If the speed limit is 80 kilometers per hour, and Elena's mom was driving 75 miles per hour, was she speeding? By how much?

4.3: Veterinary Weights

A veterinarian uses weights in kilograms to figure out what dosages of medicines to

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prescribe for animals. For every 10 kilograms, there are 22 pounds.

1. Calculate each animal's weight in kilograms. Explain or show your reasoning. If you get stuck, consider drawing a double number line or table.

a. Fido the Labrador weighs 88 pounds.

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- b. Spot the Beagle weighs 33 pounds.
- c. Bella the Chihuahua weighs $5\frac{1}{2}$ pounds.

2. A certain medication says it can only be given to animals over 25 kilograms. How much is this in pounds?

4.4: Cooking with a Tablespoon

Diego is trying to follow a recipe, but he cannot find any measuring cups! He only has a tablespoon. In the cookbook, it says that 1 cup equals 16 tablespoons.

1. How could Diego use the tablespoon to measure out these ingredients?

- a. $\frac{1}{2}$ cup almonds b. $1\frac{1}{4}$ cup of oatmeal c. $2\frac{3}{4}$ cup of flour

2. Diego also adds the following ingredients. How many cups of each did he use?

- a. 28 tablespoons of sugar b. 6 tablespoons of cocoa powder

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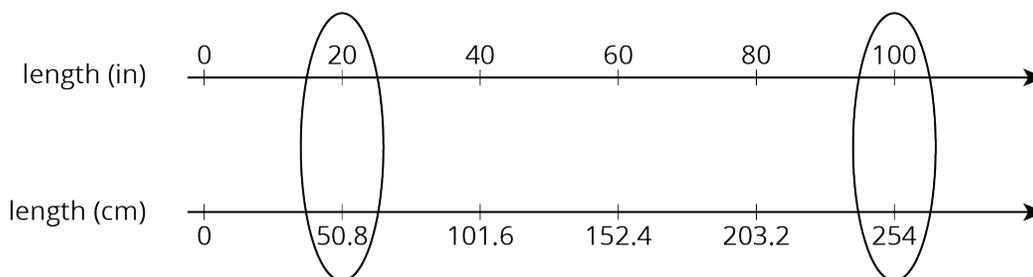
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Lesson 4 Summary

When we measure something in two different units, the measurements form an equivalent ratio. We can reason with these equivalent ratios to convert measurements from one unit to another.

Suppose you cut off 20 inches of hair. Your Canadian friend asks how many centimeters of hair that was. Since 100 inches equal 254 centimeters, we can use equivalent ratios to find out how many centimeters equal 20 inches.

Using a double number line:



Using a table:

length (in)	length (cm)
100	254
1	2.54
20	50.8

One quick way to solve the problem is to start by finding out how many centimeters are in 1 inch. We can then multiply 2.54 and 20 to find that 20 inches equal 50.8 centimeters.

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Unit 3, Lesson 4: Converting Units

1. Priya's family exchanged 250 dollars for 4,250 pesos. Priya bought a sweater for 510 pesos. How many dollars did the sweater cost?

pesos	dollars
4,250	250
	25
	1
	3
510	

2. There are 3,785 milliliters in 1 gallon, and there are 4 quarts in 1 gallon. For each question, explain or show your reasoning.
- How many milliliters are in 3 gallons?
 - How many milliliters are in 1 quart?
3. Lin knows that there are 4 quarts in a gallon. She wants to convert 6 quarts to gallons, but cannot decide if she should multiply 6 by 4 or divide 6 by 4 to find her answer. What should she do? Explain or show your reasoning. If you get stuck, consider drawing a double number line or using a table.
4. Tyler has a baseball bat that weighs 28 ounces. Find this weight in kilograms and in grams. (Note: 1 kilogram \approx 35 ounces)
5. Identify whether each unit measures length, volume, or weight (or mass).

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a. Mile

e. Liter

i. Kilogram

b. Cup

f. Gram

j. Teaspoon

c. Pound

g. Pint

k. Milliliter

d. Centimeter

h. Yard

(from Unit 3, Lesson 1)

6. A recipe for trail mix uses 7 ounces of almonds with 5 ounces of raisins. (Almonds and raisins are the only ingredients.) How many ounces of almonds would be in a one-pound bag of this trail mix? Explain or show your reasoning.

(from Unit 2, Lesson 11)

7. An ant can travel at a constant speed of 980 inches every 5 minutes.

a. How far does the ant travel in 1 minute?

b. At this rate, how far can the ant travel in 7 minutes?

(from Unit 2, Lesson 9)

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Unit 3, Lesson 5: Comparing Speeds and Prices

Let's compare some speeds and some prices.

5.1: Closest Quotient

Is the value of each expression closer to $\frac{1}{2}$, 1, or $1\frac{1}{2}$?

1. $20 \div 18$

2. $9 \div 20$

3. $7 \div 5$

5.2: More Treadmills

Some students did treadmill workouts, each one running at a constant speed. Answer the questions about their workouts. Explain or show your reasoning.

- Tyler ran 4,200 meters in 30 minutes.
- Kiran ran 6,300 meters in $\frac{1}{2}$ hour.
- Mai ran 6.3 kilometers in 45 minutes.

1. What is the same about the workouts done by:

- a. Tyler and Kiran?
- b. Kiran and Mai?
- c. Mai and Tyler?

2. At what rate did each of them run?

3. How far did Mai run in her first 30 minutes on the treadmill?

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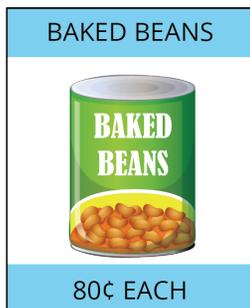
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Are you ready for more?

Tyler and Kiran each started running at a constant speed at the same time. Tyler ran 4,200 meters in 30 minutes and Kiran ran 6,300 meters in $\frac{1}{2}$ hour. Eventually, Kiran ran 1 kilometer more than Tyler. How much time did it take for this to happen?

5.3: The Best Deal on Beans

Four different stores posted ads about special sales on 15-oz cans of baked beans.



1. Which store is offering the best deal?
Explain your reasoning.

2. The last store listed is also selling 28-oz cans of baked beans for \$1.40 each. How does that price compare to the other prices?

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Lesson 5 Summary

Diego ran 3 kilometers in 20 minutes. Andre ran 2,550 meters in 17 minutes. Who ran faster? Since neither their distances nor their times are the same, we have two possible strategies:

- Find the time each person took to travel the *same distance*. The person who traveled that distance in less time is faster.
- Find the distance each person traveled in the *same time*. The person who traveled a longer distance in the same amount of time is faster.

It is often helpful to compare distances traveled in *1 unit* of time (1 minute, for example), which means finding the speed such as meters per minute.

Let's compare Diego and Andre's speeds in meters per minute.

distance (meters)	time (minutes)
3,000	20
1,500	10
150	1

distance (meters)	time (minutes)
2,550	17
150	1

Both Diego and Andre ran 150 meters per minute, so they ran at the same speed.

Finding ratios that tell us how much of quantity *A* per 1 unit of quantity *B* is an efficient way to compare rates in different situations. Here are some familiar examples:

- Car speeds in *miles per hour*.
- Fruit and vegetable prices in *dollars per pound*.

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Unit 3, Lesson 5: Comparing Speeds and Prices

1. Mai and Priya were on scooters. Mai traveled 15 meters in 6 seconds. Priya travels 22 meters in 10 seconds. Who was moving faster? Explain your reasoning.

2. Here are the prices for cans of juice that are the same brand and the same size at different stores. Which store offers the best deal? Explain your reasoning.

Store X: 4 cans for \$2.48

Store Y: 5 cans for \$3.00

Store Z: 59 cents per can

3. Costs of homes can be very different in different parts of the United States.

a. A 450-square-foot apartment in New York City costs \$540,000. What is the price per square foot? Explain or show your reasoning.

b. A 2,100-square-foot home in Cheyenne, Wyoming, costs \$110 per square foot. How much does this home cost? Explain or show your reasoning.

4. There are 33.8 fluid ounces in a liter. There are 128 fluid ounces in a gallon. About how many liters are in a gallon?

a. 2

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b. 3

c. 4

d. 5

Is your estimate larger or smaller than the actual number of liters in a gallon? Explain how you know.

(from Unit 3, Lesson 4)

5. Diego is 165 cm tall. Andre is 1.7 m tall. Who is taller, Diego or Andre? Explain your reasoning.

(from Unit 3, Lesson 3)

6. Name an object that could be about the same length as each measurement.

a. 4 inches

e. 6 centimeters

b. 6 feet

f. 2 millimeters

c. 1 meter

g. 3 kilometers

d. 5 yards

(from Unit 3, Lesson 2)

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Unit 3, Lesson 6: Interpreting Rates

Let's explore unit rates.

6.1: Something per Something

1. Think of two things you have heard described in terms of "something per something."

2. Share your ideas with your group, and listen to everyone else's idea. Make a group list of all unique ideas. Be prepared to share these with the class.

6.2: Cooking Oatmeal

Priya, Han, Lin, and Diego are all on a camping trip with their families. The first morning, Priya and Han make oatmeal for the group. The instructions for a large batch say, "Bring 15 cups of water to a boil, and then add 6 cups of oats."

- Priya says, "The ratio of the cups of oats to the cups of water is 6 : 15. That's 0.4 cups of oats per cup of water."
- Han says, "The ratio of the cups of water to the cups of oats is 15 : 6. That's 2.5 cups of water per cup of oats."

1. Who is correct? Explain your reasoning. If you get stuck, consider using the table.

water (cups)	oats (cups)
15	6
1	
	1

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2. The next weekend after the camping trip, Lin and Diego each decide to cook a large batch of oatmeal to have breakfasts ready for the whole week.
- a. Lin decides to cook 5 cups of oats. How many cups of water should she boil?
- b. Diego boils 10 cups of water. How many cups of oats should he add into the water?
3. Did you use Priya's rate (0.4 cups of oats per cup of water) or Han's rate (2.5 cups of water per cup of oats) to help you answer each of the previous two questions? Why?

6.3: Cheesecake, Milk, and Raffle Tickets

For each situation, find the **unit rates**.

1. A cheesecake recipe says, "Mix 12 oz of cream cheese with 15 oz of sugar."
- a. How many ounces of cream cheese are there for every ounce of sugar?
- b. How many ounces of sugar is that for every ounce of cream cheese?
2. Mai's family drinks a total of 10 gallons of milk every 6 weeks.
- a. How many gallons of milk does the family drink per week?
- b. How many weeks does it take the family to consume 1 gallon of milk?

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3. Tyler paid \$16 for 4 raffle tickets.

a. What is the price per ticket?

b. How many tickets is that per dollar?

4. For each problem, decide which unit rate from the previous situations you prefer to use. Next, solve the problem, and show your thinking.

a. If Lin wants to make extra cheesecake filling, how much cream cheese will she need to mix with 35 ounces of sugar?

b. How many weeks will it take Mai's family to finish 3 gallons of milk?

c. How much would all 1,000 raffle tickets cost?

Are you ready for more?

Write a "deal" on tickets for Tyler's raffle that sounds good, but is actually a little worse than just buying tickets at the normal price.

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Lesson 6 Summary

Suppose a farm lets us pick 2 pounds of blueberries for 5 dollars. We can say:

blueberries (pounds)	price (dollars)
2	5
1	$\frac{5}{2}$
$\frac{2}{5}$	1

- We get $\frac{2}{5}$ pound of blueberries per dollar.
- The blueberries cost $\frac{5}{2}$ dollars per pound.

The “cost per pound” and the “number of pounds per dollar” are the two *unit rates* for this situation.

A **unit rate** tells us how much of one quantity for 1 of the other quantity. Each of these numbers is useful in the right situation.

If we want to find out how much 8 pounds of blueberries will cost, it helps to know how much 1 pound of blueberries will cost.

blueberries (pounds)	price (dollars)
1	$\frac{5}{2}$
8	$8 \cdot \frac{5}{2}$

If we want to find out how many pounds we can buy for 10 dollars, it helps to know how many pounds we can buy for 1 dollar.

blueberries (pounds)	price (dollars)
$\frac{2}{5}$	1
$10 \cdot \frac{2}{5}$	10

Which unit rate is most useful depends on what question we want to answer, so be ready to find either one!

Lesson 6 Glossary Terms

- unit rate

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Unit 3, Lesson 6: Interpreting Rates

1. A pink paint mixture uses 4 cups of white paint for every 3 cups of red paint.

The table shows different quantities of red and white paint for the same shade of pink. Complete the table.

white paint (cups)	red paint (cups)
4	3
	1
1	
	4
5	

2. A farm lets you pick 3 pints of raspberries for \$12.00.

- What is the cost per pint?
- How many pints do you get per dollar?
- At this rate, how many pints can you afford for \$20.00?
- At this rate, how much will 8 pints of raspberries cost?

3. Han and Tyler are following a polenta recipe that uses 5 cups of water for every 2 cups of cornmeal.

- Han says, "I am using 3 cups of water. I will need $1\frac{1}{5}$ cups of cornmeal."
- Tyler says, "I am using 3 cups of cornmeal. I will need $7\frac{1}{2}$ cups of water."

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Do you agree with either of them? Explain your reasoning.

4. A large art project requires enough paint to cover 1,750 square feet. Each gallon of paint can cover 350 square feet. Each square foot requires $\frac{1}{350}$ of a gallon of paint.

Andre thinks he should use the rate $\frac{1}{350}$ gallons of paint per square foot to find how much paint they need. Do you agree with Andre? Explain or show your reasoning.

5. Andre types 208 words in 4 minutes. Noah types 342 words in 6 minutes. Who types faster? Explain your reasoning.

(from Unit 3, Lesson 5)

6. A corn vendor at a farmer's market was selling a bag of 8 ears of corn for \$2.56. Another vendor was selling a bag of 12 for \$4.32. Which bag is the better deal? Explain or show your reasoning.

(from Unit 3, Lesson 5)

7. A soccer field is 100 meters long. What could be its length in yards?

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A. 33.3

B. 91

C. 100

D. 109

(from Unit 3, Lesson 3)

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Unit 3, Lesson 7: Equivalent Ratios Have the Same Unit Rates

Let's revisit equivalent ratios.

7.1: Which One Doesn't Belong: Comparing Speeds

Which one doesn't belong? Be prepared to explain your reasoning.

A. 5 miles in 15 minutes

C. 20 miles per hour

B. 3 minutes per mile

D. 32 kilometers per hour

7.2: Price of Burritos

1. Two burritos cost \$14.00. Complete the table to show the cost for 4, 5, and 10 burritos at that rate. Next, find the cost for a single burrito in each case.

number of burritos	cost in dollars	unit price (dollars per burrito)
2	14.00	
4		
5		
10		
b		

2. What do you notice about the values in this table?

3. Noah bought b burritos and paid c dollars. Lin bought twice as many burritos as Noah and paid twice the cost he did. How much did Lin pay per burrito?

	number of burritos	cost in dollars	unit price (dollars per burrito)
Noah	b	c	$\frac{c}{b}$
Lin	$2 \cdot b$	$2 \cdot c$	

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4. Explain why, if you can buy b burritos for c dollars, or buy $2 \cdot b$ burritos for $2 \cdot c$ dollars, the cost per item is the same in either case.

7.3: Making Bracelets

1. Complete the table. Then, explain the strategy you used to do so.

time in hours	number of bracelets	speed (bracelets per hour)
2		6
5		6
7		6
	66	6
	100	6



2. Here is a partially filled table from an earlier activity. Use the same strategy you used for the bracelet problem to complete this table.

number of burritos	cost in dollars	unit price (dollars per burrito)
	14.00	7.00
	28.00	7.00
5		7.00
10		7.00

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3. Next, compare your results with those in the first table in the previous activity. Do they match? Explain why or why not.

7.4: How Much Applesauce?

It takes 4 pounds of apples to make 6 cups of applesauce.

1. At this rate, how much applesauce can you make with:

a. 7 pounds of apples?

b. 10 pounds of apples?

2. How many pounds of apples would you need to make:

a. 9 cups of applesauce?

b. 20 cups of applesauce?

pounds of apples	cups of applesauce
4	6
7	
10	
	9
	20

Are you ready for more?

- Jada eats 2 scoops of ice cream in 5 minutes. Noah eats 3 scoops of ice cream in 5 minutes. How long does it take them to eat 1 scoop of ice cream working together (if they continue eating ice cream at the same rate they do individually)?
- The garden hose at Andre's house can fill a 5-gallon bucket in 2 minutes. The hose at his next-door neighbor's house can fill a 10-gallon bucket in 8 minutes. If they use both their garden hoses at the same time, and the hoses continue working at the same rate they did when filling a bucket, how long will it take to fill a 750-gallon pool?

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Lesson 7 Summary

The table shows different amounts of apples selling at the same rate, which means all of the ratios in the table are equivalent. In each case, we can find the *unit price* in dollars per pound by dividing the price by the number of pounds.

apples (pounds)	price (dollars)	unit price (dollars per pound)
4	10	$10 \div 4 = 2.50$
8	20	$20 \div 8 = 2.50$
20	50	$50 \div 20 = 2.50$

The unit price is always the same. Whether we buy 10 pounds of apples for 4 dollars or 20 pounds of apples for 8 dollars, the apples cost 2.50 dollars per pound.

We can also find the number of pounds of apples we can buy per dollar by dividing the number of pounds by the price.

apples (pounds)	price (dollars)	pounds per dollar
4	10	$4 \div 10 = 0.4$
8	20	$8 \div 20 = 0.4$
20	50	$20 \div 50 = 0.4$

The number of pounds we can buy for a dollar is the same as well! Whether we buy 10 pounds of apples for 4 dollars or 20 pounds of apples for 8 dollars, we are getting 0.4 pounds per dollar.

This is true in all contexts: when two ratios are equivalent, the two unit rates will always be equal.

quantity x	quantity y	unit rate 1	unit rate 2
a	b	$\frac{a}{b}$	$\frac{b}{a}$
$s \cdot a$	$s \cdot b$	$\frac{s \cdot a}{s \cdot b} = \frac{a}{b}$	$\frac{s \cdot b}{s \cdot a} = \frac{b}{a}$

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Unit 3, Lesson 7: Equivalent Ratios Have the Same Unit Rates

1. A car travels 55 miles per hour for 2 hours. Complete the table.

time (hours)	distance (miles)	miles per hour
1	55	55
$\frac{1}{2}$		
$1\frac{1}{2}$		
	110	

2. The table shows the amounts of onions and tomatoes in different-sized batches of a salsa recipe.

Elena notices that if she takes the number in the tomatoes column and divides it by the corresponding number in the onions column, she always gets the same result.

What is the meaning of the number that Elena has calculated?

onions (ounces)	tomatoes (ounces)
2	16
4	32
6	48

3. A restaurant is offering 2 specials: 10 burritos for \$12, or 6 burritos for \$7.50. Noah needs 60 burritos for his party. Should he buy 6 orders of the 10-burrito special or 10 orders of the 6-burrito special? Explain your reasoning.

4. Complete the table so that the cost per banana remains the same.

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number of bananas	cost in dollars	unit price (dollars per banana)
4		0.50
6		0.50
7		0.50
10		0.50
	10.00	0.50
	16.50	0.50

5. Two planes travel at a constant speed. Plane A travels 2,800 miles in 5 hours. Plane B travels 3,885 miles in 7 hours. Which plane is faster? Explain your reasoning.

(from Unit 3, Lesson 5)

6. A car has 15 gallons of gas in its tank. The car travels 35 miles per gallon of gas. It uses $\frac{1}{35}$ of a gallon of gas to go 1 mile.

a. How far can the car travel with 15 gallons?
Show your reasoning.

b. How much gas does the car use to go 100 miles? Show your reasoning.

(from Unit 3, Lesson 6)

7. A box of cereal weighs 600 grams. How much is this weight in pounds? Explain or show your

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reasoning. (Note: 1 kilogram = 2.2 pounds)

(from Unit 3, Lesson 4)

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Unit 3, Lesson 8: More about Constant Speed

Let's investigate constant speed some more.

8.1: Back on the Treadmill Again

While training for a race, Andre's dad ran 12 miles in 75 minutes on a treadmill. If he runs at that rate:

1. How long would it take him to run 8 miles?
2. How far could he run in 30 minutes?

8.2: Picnics on the Rail Trail



Kiran and Clare live 24 miles away from each other along a rail trail. One Saturday, the two friends started walking toward each other along the trail at 8:00 a.m. with a plan to have a picnic when they meet.

Kiran walks at a **speed** of 3 miles per hour while Clare walks 3.4 miles per hour.

1. After one hour, how far apart will they be?
2. Make a table showing how far apart the two friends are after 0 hours, 1 hour, 2 hours, and 3 hours.

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3. At what time will the two friends meet and have their picnic?

4. Kiran says “If I walk 3 miles per hour toward you, and you walk 3.4 miles per hour toward me, it’s the same as if you stay put and I jog 6.4 miles per hour.” What do you think Kiran means by this? Is he correct?

5. Several months later, they both set out at 8:00 a.m. again, this time with Kiran jogging and Clare still walking at 3.4 miles per hour. This time, they meet at 10:30 a.m. How fast was Kiran jogging?

Are you ready for more?

1. On his trip to meet Clare, Kiran brought his dog with him. At the same time Kiran and Clare started walking, the dog started running 6 miles per hour. When it got to Clare it turned around and ran back to Kiran. When it got to Kiran, it turned around and ran back to Clare, and continued running in this fashion until Kiran and Clare met. How far did the dog run?

2. The next Saturday, the two friends leave at the same time again, and Kiran jogs twice as fast as Clare walks. Where on the rail trail do Kiran and Clare meet?

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8.3: Swimming and Biking

Jada bikes 2 miles in 12 minutes. Jada's cousin swims 1 mile in 24 minutes.

1. Who was moving faster? How much faster?

2. One day Jada and her cousin line up on the end of a swimming pier on the edge of a lake. At the same time, they start swimming and biking in opposite directions.

a. How far apart will they be after 15 minutes?

b. How long will it take them to be 5 miles apart?

Lesson 8 Summary

When two objects are each moving at a constant speed and their distance-to-time ratios are equivalent, we say that they are moving at the *same speed*. If their time-distance ratios are not equivalent, they are not moving at the same speed.

We describe **speed** in units of distance per unit of time, like *miles per hour* or *meters per second*.

- A snail that crawls 5 centimeters in 2 minutes is traveling at a rate of 2.5 centimeters per minute.
- A toddler that walks 9 feet in 6 seconds is traveling at a rate of 1.5 feet per second.
- A cyclist who bikes 20 kilometers in 2 hours is traveling at a rate of 10 kilometers per hour.

We can also use **pace** to describe distance and time. We measure pace in units such as *hours per mile* or *seconds per meter*.

- A snail that crawls 5 centimeters in 2 minutes has a pace of 0.4 minutes per centimeter.
- A toddler walking 9 feet in 6 seconds has a pace of $\frac{2}{3}$ seconds per foot.
- A cyclist who bikes 20 kilometers in 2 hours has a pace of 0.1 hours per kilometer.

Speed and pace are reciprocals. Both can be used to compare whether one object is

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moving faster or slower than another object.

- An object with the higher speed is *faster* than one with a lower speed because the former travels a greater distance in the same amount of time.
- An object with the greater pace is *slower* than one with a smaller pace because the former takes more time to travel the same distance.

Because speed is a *rate per 1 unit of time* for ratios that relate distance and time, we can multiply the amount of time traveled by the speed to find the distance traveled.

time (minutes)	distance (centimeters)
2	5
1	2.5
4	$4 \cdot (2.5)$

Lesson 8 Glossary Terms

- pace
- speed

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Unit 3, Lesson 8: More about Constant Speed

1. A kangaroo hops 2 kilometers in 3 minutes. At this rate:

- a. How long does it take the kangaroo to travel 5 kilometers?
- b. How far does the kangaroo travel in 2 minutes?

2. Mai runs around a 400-meter track at a constant speed of 250 meters per minute. How many minutes does it take Mai to complete 4 laps of the track? Explain or show your reasoning.

3. At 10:00 a.m., Han and Tyler both started running toward each other from opposite ends of a 10-mile path along a river. Han runs at a pace of 12 minutes per mile. Tyler runs at a pace of 15 minutes per mile.

a. How far does Han run after a half hour? After an hour?

b. Do Han and Tyler meet on the path within 1 hour? Explain or show your reasoning.

4. Two skateboarders start a race at the same time. Skateboarder A travels at a steady rate of 15 feet per

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second. Skateboarder B travels at a steady rate of 22 feet per second. After 4 minutes, how much farther will Skateboarder B have traveled? Explain your reasoning.

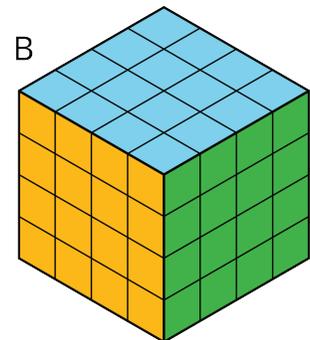
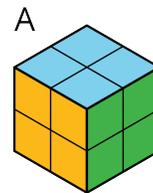
(from Unit 2, Lesson 16)

5. There are 4 tablespoons in $\frac{1}{4}$ cup. There are 2 cups in 1 pint. How many tablespoons are there in 1 pint? If you get stuck, consider drawing a double number line or making a table.

(from Unit 3, Lesson 4)

6. Two larger cubes are made out of unit cubes. Cube A is 2 by 2 by 2. Cube B is 4 by 4 by 4. The side length of Cube B is twice that of Cube A.

a. Is the surface area of Cube B also twice that of Cube A? Explain or show your reasoning.



b. Is the volume of Cube B also twice that of Cube A? Explain or show your reasoning.

(from Unit 1, Lesson 12)

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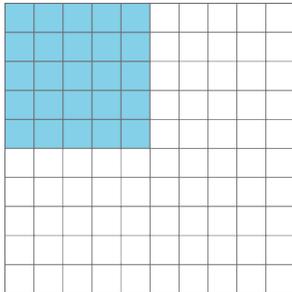
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Unit 3, Lesson 9: Solving Rate Problems

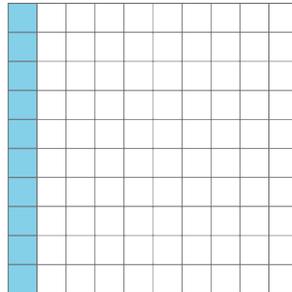
Let's use unit rates like a pro.

9.1: Grid of 100

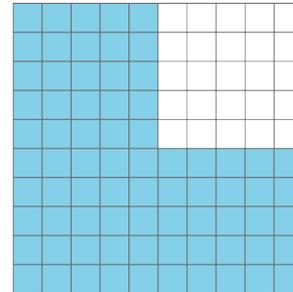
How much is shaded in each one?



A



B



C

9.2: Card Sort: Is it a Deal?

Your teacher will give you a set of cards showing different offers.

1. Find card A and work with your partner to decide whether the offer on card A is a good deal. Explain or show your reasoning.
2. Next, split cards B–E so you and your partner each have two.
 - a. Decide individually if your two cards are good deals. Explain your reasoning.
 - b. For each of your cards, explain to your partner if you think it is a good deal and why. Listen to your partner's explanations for their cards. If you disagree, explain your thinking.
 - c. Revise any decisions about your cards based on the feedback from your partner.

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3. When you and your partner are in agreement about cards B–E, place all the cards you think are a good deal in one stack and all the cards you think are a bad deal in another stack. Be prepared to explain your reasoning.

Are you ready for more?

Time to make your own deal! Read the information on card F and then decide what you would charge if you were the clerk. When your teacher signals, trade cards with another group and decide whether or not you would take the other group's offer.

Keep in mind that you may offer a fair deal or an unfair deal, but the goal is to set a price close enough to the value it should be that the group they cannot immediately tell if the deal you offer is a good one.

9.3: The Fastest of All

Wild animals from around the world wanted to hold an athletic competition, but no one would let them on an airplane. They decided to just measure how far each animal could sprint in one minute and send the results to you to decide the winner.

animal	sprint distance
cougar	1,408 yards
antelope	1 mile
hare	49,632 inches
kangaroo	1,073 meters
ostrich	1.15 kilometers
coyote	3,773 feet

You look up the following information about converting units of length:

$$1 \text{ inch} = 2.54 \text{ centimeters}$$

1. Which animal sprinted the farthest?
2. What are the place rankings for all of the animals?

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Lesson 9 Summary

Sometimes we can find and use more than one unit rate to solve a problem.

Suppose a grocery store is having a sale on shredded cheese. A small bag that holds 8 ounces is sold for \$2. A large bag that holds 2 kilograms is sold for \$16. How do you know which is a better deal?

Here are two different ways to solve this problem:

Compare dollars per kilogram.

- The large bag costs \$8 per kilogram, because $16 \div 2 = 8$.
- The small bag holds $\frac{1}{2}$ pound of cheese, because there are 16 ounces in 1 pound, and $8 \div 16 = \frac{1}{2}$.

The small bag costs \$4 per pound, because $2 \div \frac{1}{2} = 4$. This is about \$8.80 per kilogram, because there are about 2.2 pounds in 1 kilogram, and $4.00 \cdot 2.2 = 8.80$.

The large bag is a better deal, because it costs less money for the same amount of cheese.

Another way to solve the problem would be to compare the unit prices of each bag in dollars per ounce. Try it!

Compare ounces per dollar.

- With the small bag, we get 4 ounces per dollar, because $8 \div 2 = 4$.
- The large bag holds 2,000 grams of cheese. There are 1,000 grams in 1 kilogram, and $2 \cdot 1,000 = 2,000$. This means 125 grams per dollar, because $2,000 \div 16 = 125$.

There are about 28.35 grams in 1 ounce, and $125 \div 28.35 \approx 4.4$, so this is about 4.4 ounces per dollar.

The large bag is a better deal, because you get more cheese for the same amount of money.

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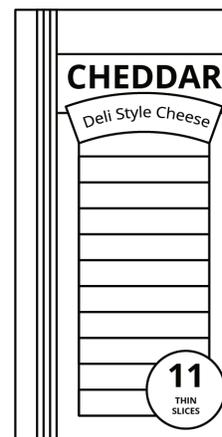
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Unit 3, Lesson 9: Solving Rate Problems

1. This package of sliced cheese costs \$2.97.

How much would a package with 18 slices cost at the same price per slice? Explain or show your reasoning.



2. A copy machine can print 480 copies every 4 minutes. For each question, explain or show your reasoning.

a. How many copies can it print in 10 minutes?

b. A teacher printed 720 copies. How long did it take to print?

3. Order these objects from heaviest to lightest. (Note: 1 pound = 16 ounces, 1 kilogram \approx 2.2 pounds, and 1 ton = 2,000 pounds)

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item	weight
school bus	9 tons
horse	1,100 pounds
elephant	5,500 kilograms
grand piano	15,840 ounces

4. Andre sometimes mows lawns on the weekend to make extra money. Two weeks ago, he mowed a neighbor's lawn for $\frac{1}{2}$ hour and earned \$10. Last week, he mowed his uncle's lawn for $\frac{3}{2}$ hours and earned \$30. This week, he mowed the lawn of a community center for 2 hours and earned \$30.

Which jobs paid better than others? Explain your reasoning.

(from Unit 3, Lesson 5)

5. Calculate and express your answer in decimal form.

a. $\frac{1}{2} \cdot 17$

c. $(0.2) \cdot 40$

b. $\frac{3}{4} \cdot 200$

d. $(0.25) \cdot 60$

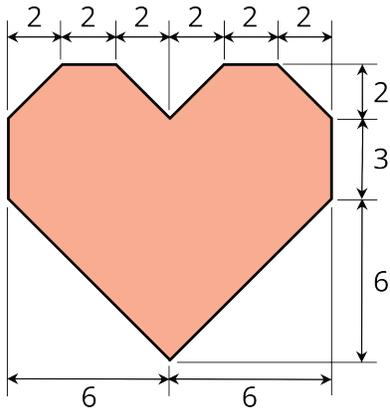
(from Unit 3, Lesson 1)

6. a. Decompose this polygon so that its area can be calculated. All measurements are in centimeters.

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(from Unit 1, Lesson 11)

b. Calculate its area. Organize your work so that it can be followed by others.

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Unit 3, Lesson 10: What Are Percentages?

Let's learn about percentages.

10.1: Dollars and Cents

Find each answer mentally.

1. A sticker costs 25 cents. How many dollars is that?
2. A pen costs 1.50 dollars. How many cents is that?
3. How many cents are in one dollar?
4. How many dollars are in one cent?

10.2: Coins

1. Complete the table to show the values of these U.S. coins.



coin	penny	nickel	dime	quarter	half dollar	dollar
value (cents)						

The value of a quarter is 25% of the value of a dollar because there are 25 cents for every 100 cents.

1 Quarter

1 Dollar

2. Write the name of the coin that matches each expression.

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1. The coins in Jada's pocket are worth 75% of a dollar. How much are they worth (in dollars)?

3. Elena has 3 quarters and 5 dimes. What percentage of a dollar does she have?

2. The coins in Diego's pocket are worth 150% of a dollar. How much are they worth (in dollars)?

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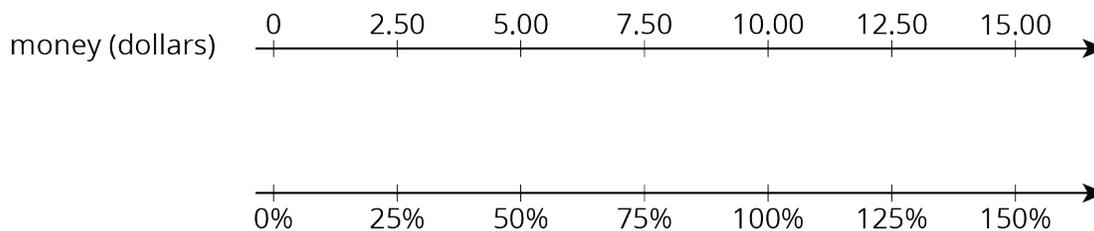
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Lesson 10 Summary

A **percentage** is a *rate per 100*.

We can find percentages of \$10 using a double number line where 10 and 100% are aligned, as shown here:



Looking at the double number line, we can see that \$5.00 is 50% of \$10.00 and that \$12.50 is 125% of \$10.00.

Lesson 10 Glossary Terms

- percent
- percentage

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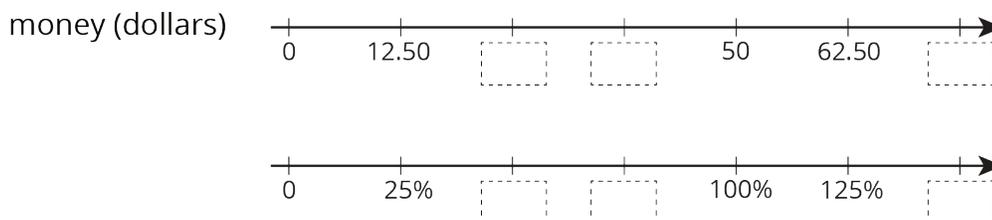
Unit 3, Lesson 10: What Are Percentages?

- What percentage of a dollar is the value of each coin combination?
 - 4 dimes
 - 1 nickel and 3 pennies
 - 5 quarters and 1 dime
- List three different combinations of coins, each with a value of 30% of a dollar.
 - List two different combinations of coins, each with a value of 140% of a dollar.
- The United States government used to make coins of many different values. For each coin, state its worth as a percentage of \$1.



- | | | |
|-----------------------|---------------------|--------|
| a. $\frac{1}{2}$ cent | c. 20 cents | e. \$5 |
| b. 3 cents | d. $\$2\frac{1}{2}$ | |

- Complete the double number to line show percentages of \$50.



- Elena bought 8 tokens for \$4.40. At this rate:

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a. How many tokens could she buy with \$6.05?

b. How much do 19 tokens cost?

(from Unit 3, Lesson 9)

6. A snail travels 10 cm in 4 minutes. At this rate:

a. How long will it take the snail to travel 24 cm?

b. How far does the snail travel in 6 minutes?

(from Unit 3, Lesson 8)

7. a. 3 tacos cost \$18. Complete the table to show the cost of 4, 5, and 6 tacos at the same rate.

number of tacos	cost in dollars	rate in dollars per taco
3	18	
4		
5		
6		

b. If you buy t tacos for c dollars, what is the unit rate?

(from Unit 3, Lesson 7)

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Unit 3, Lesson 11: Percentages and Double Number Lines

Let's use double number lines to represent percentages.

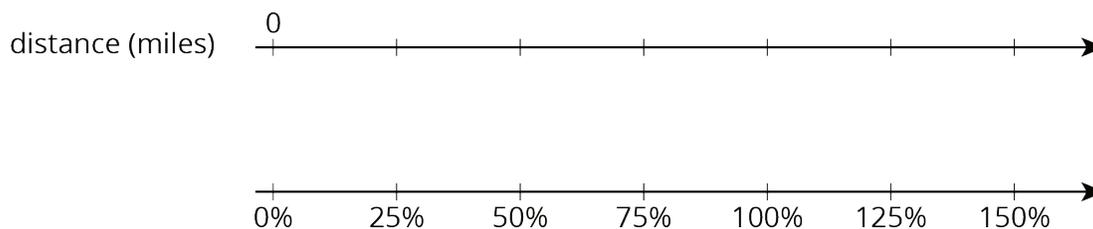
11.1: Fundraising Goal

Each of three friends—Lin, Jada, and Andre—had the goal of raising \$40. How much money did each person raise? Be prepared to explain your reasoning.

1. Lin raised 100% of her goal.
2. Jada raised 50% of her goal.
3. Andre raised 150% of his goal.

11.2: Three-Day Biking Trip

Elena biked 8 miles on Saturday. Use the double number line to answer the questions. Be prepared to explain your reasoning.



1. What is 100% of her Saturday distance?
2. On Sunday, she biked 75% of her Saturday distance. How far was that?
3. On Monday, she biked 125% of her Saturday distance. How far was that?

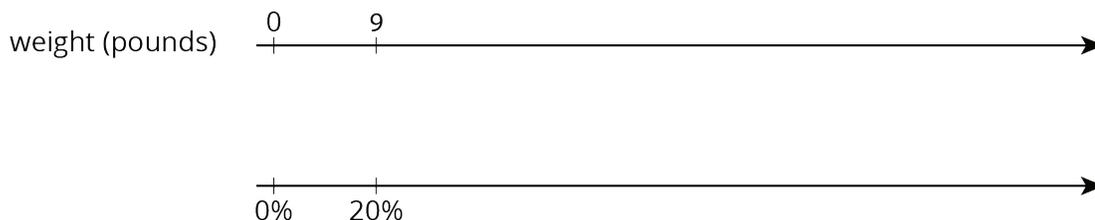
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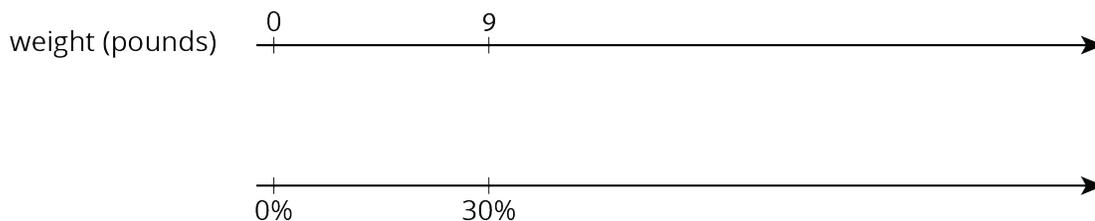
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11.3: Puppies Grow Up

1. Jada has a new puppy that weighs 9 pounds. The vet says that the puppy is now at about 20% of its adult weight. What will be the adult weight of the puppy?



2. Andre also has a puppy that weighs 9 pounds. The vet says that this puppy is now at about 30% of its adult weight. What will be the adult weight of Andre's puppy?



3. What is the same about Jada and Andre's puppies? What is different?

Are you ready for more?

A loaf of bread costs \$3.00 today. The same size loaf cost 20 cents in 1955.

1. What percentage of today's price did someone in 1955 pay for bread?
2. A job pays \$10.00 an hour today. If the same percentage applies to income as well, how much would that job have paid in 1955?

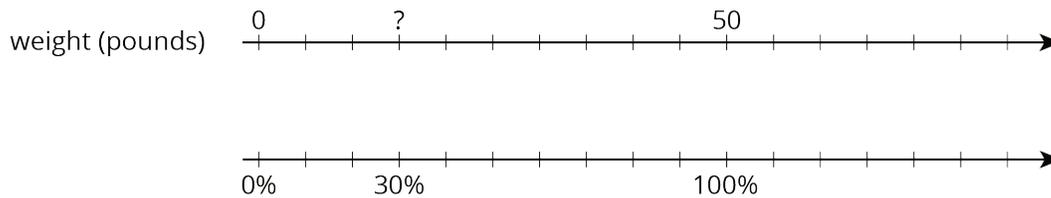
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Lesson 11 Summary

We can use a double number line to solve problems about percentages. For example, what is 30% of 50 pounds? We can draw a double number line like this:



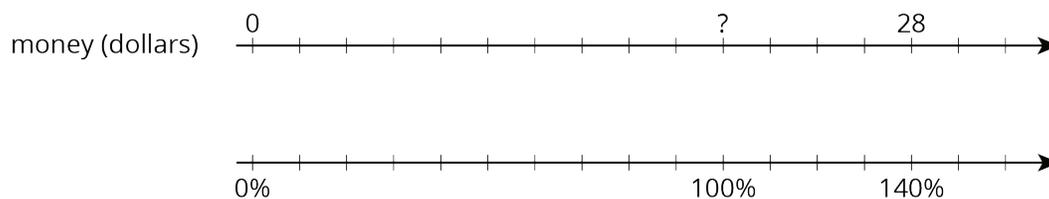
We divide the distance between 0% and 100% and that between 0 and 50 pounds into ten equal parts. We label the tick marks on the top line by counting by 5s ($50 \div 10 = 5$) and on the bottom line counting by 10% ($100 \div 10 = 10$). We can then see that 30% of 50 pounds is 15 pounds.

We can also use a table to solve this problem.

weight (pounds)	percentage
50	100
5	10
15	30

Annotations: On the left, green arrows point from 50 to 5 (labeled $\cdot \frac{1}{10}$) and from 5 to 15 (labeled $\cdot 3$). On the right, green arrows point from 100 to 10 (labeled $\cdot \frac{1}{10}$) and from 10 to 30 (labeled $\cdot 3$).

Suppose we know that 140% of an amount is \$28. What is 100% of that amount? Let's use a double number line to find out.



We divide the distance between 0% and 140% and that between \$0 and \$28 into fourteen equal intervals. We label the tick marks on the top line by counting by 2s and on the bottom line counting by 10%. We would then see that 100% is \$20.

Or we can use a table as shown.

money (dollars)	percentage
28	140
2	10
20	100

Annotations: On the left, green arrows point from 28 to 2 (labeled $\cdot \frac{1}{14}$) and from 2 to 20 (labeled $\cdot 10$). On the right, green arrows point from 140 to 10 (labeled $\cdot \frac{1}{14}$) and from 10 to 100 (labeled $\cdot 10$).

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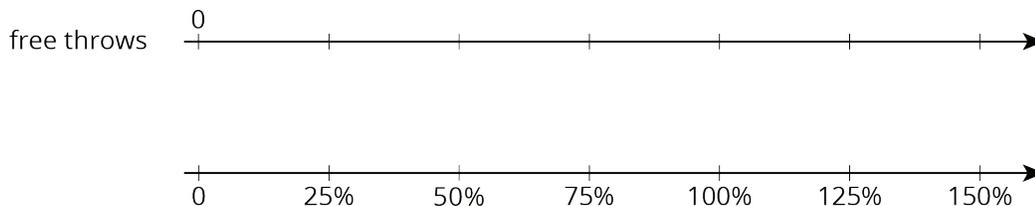
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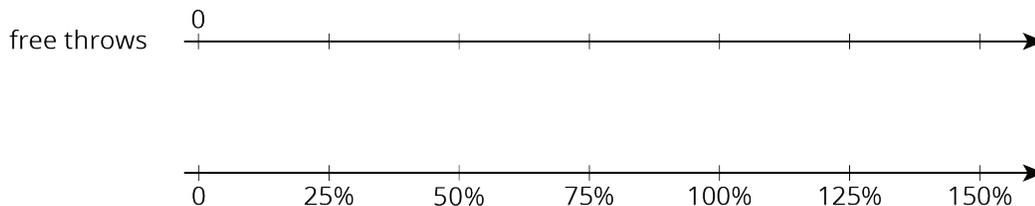
Unit 3, Lesson 11: Percentages and Double Number Lines

1. Solve each problem. If you get stuck, consider using the double number lines.

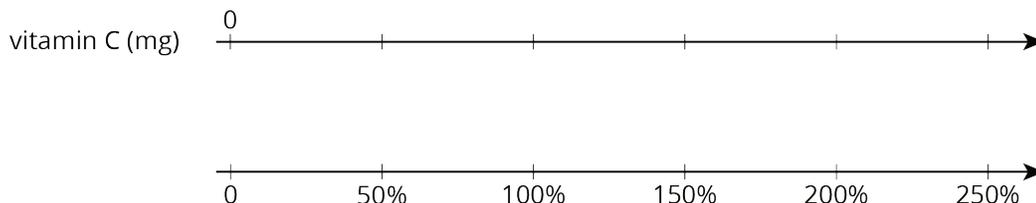
- a. During a basketball practice, Mai attempted 40 free throws and was successful on 25% of them. How many successful free throws did she make?



- b. Yesterday, Priya successfully made 12 free throws. Today, she made 150% as many. How many successful free throws did Priya make today?



2. A 16-ounce bottle of orange juice says it contains 200 milligrams of vitamin C, which is 250% of the daily recommended allowance of vitamin C for adults. What is 100% of the daily recommended allowance of vitamin C for adults?



3. At a school, 40% of the sixth-grade students said that hip-hop is their favorite kind of music. If 100 sixth-grade students prefer hip hop music, how many sixth-grade students are at the school? Explain or show your reasoning.
4. Diego has a skateboard, scooter, bike, and go-cart. He wants to know which vehicle is the fastest. A friend records how far Diego travels on each vehicle in 5 seconds. For each vehicle, Diego travels as fast as he can along a straight, level path.

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vehicle	distance traveled
skateboard	90 feet
scooter	1,020 inches
bike	4,800 centimeters
go-cart	0.03 kilometers

(from Unit 3, Lesson 9)

- a. 100 inches equal 254 centimeters. What is the distance each vehicle traveled in centimeters?
- b. Rank the vehicles in order from fastest to slowest.

5. It takes 10 pounds of potatoes to make 15 pounds of mashed potatoes. At this rate:

- a. How many pounds of mashed potatoes can they make with 15 pounds of potatoes?
- b. How many pounds of potatoes are needed to make 50 pounds of mashed potatoes?

(from Unit 3, Lesson 7)

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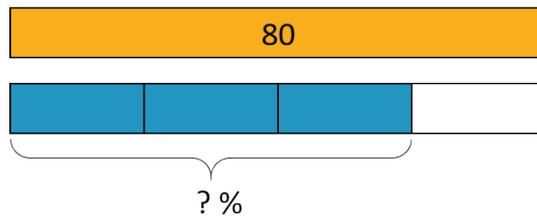
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Unit 3, Lesson 12: Percentages and Tape Diagrams

Let's use tape diagrams to understand percentages.

12.1: Notice and Wonder: Tape Diagrams

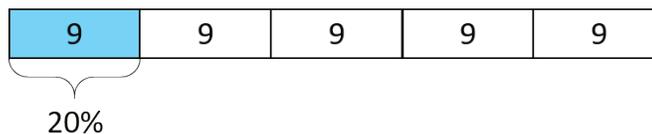
What do you notice? What do you wonder?



12.2: Revisiting Jada's Puppy

Jada has a new puppy that weighs 9 pounds. It is now at about 20% of its adult weight.

1. Here is a diagram that Jada drew about the weight of her puppy.



a. The adult weight of the puppy will be 45 pounds. How can you see that in the diagram?

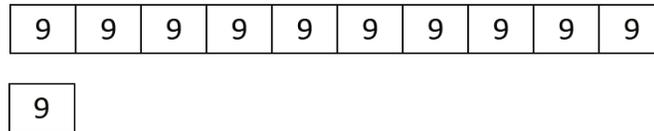
b. What fraction of its adult weight is the puppy now? How can you see that in the diagram?

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2. Jada's friend has a dog that weighs 90 pounds. Here is a diagram Jada drew that represents the weight of her friend's dog and the weight of her puppy.



- How many times greater is the dog's weight than the puppy's?
- Compare the weight of the puppy and the dog using fractions.
- Compare the weight of the puppy and the dog using percentages.

12.3: 5 Dollars

Noah has \$5.

- Elena has 40% as much as Noah. How much does Elena have?
 - Compare Elena's and Noah's money using fractions. Draw a diagram to illustrate.

- Diego has 150% as much as Noah. How much does Diego have?
 - Compare Diego's and Noah's money using fractions. Draw a diagram to illustrate.

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12.4: Staying Hydrated

During the first part of a hike, Andre drank 1.5 liters of the water he brought.

1. If this is 50% of the water he brought, how much water did he bring?
2. If he drank 80% of his water on his entire hike, how much did he drink?

Are you ready for more?

Decide if each scenario is possible.

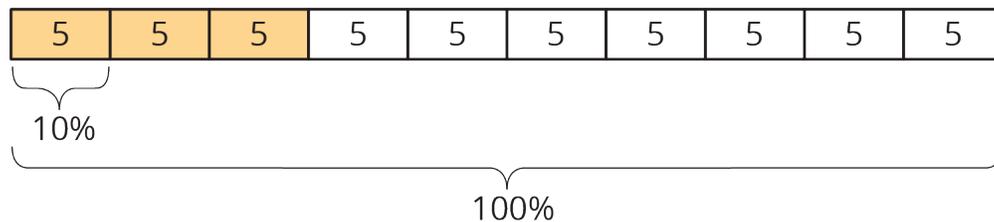
1. Andre plans to bring his dog on his next hike, along with 150% as much water as he brought on this hike.
2. Andre plans to drink 150% of the water he brought on his hike.

Lesson 12 Summary

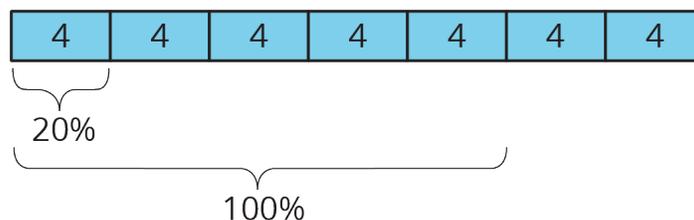
Tape diagrams can help us make sense of percentages.

Consider two problems that we solved earlier using double number lines and tables: “What is 30% of 50 pounds?” and “What is 100% of a number if 140% of it is 28?”

Here is a tape diagram that shows that 30% of 50 pounds is 15 pounds.



This diagram shows that if 140% of some number is 28, then that number must be 20.



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Unit 3, Lesson 12: Percentages and Tape Diagrams

1. Here is a tape diagram that shows how far two students walked.

Priya's distance (km)	2	2	2	2	2
Tyler's distance (km)	2	2	2	2	

- What percentage of Priya's distance did Tyler walk?
 - What percentage of Tyler's distance did Priya walk?
2. A bakery makes 40 different flavors of muffins. 25% of the flavors have chocolate as one of the ingredients. Draw a tape diagram to show how many flavors have chocolate and how many don't.
3. There are 70 students in the school band. 40% of them are sixth graders, 20% are seventh graders, and the rest are eighth graders.
- How many band members are sixth graders?
 - How many band members are seventh graders?
 - What percentage of the band members are eighth graders? Explain your reasoning.
4. Jada has a monthly budget for her cell phone bill. Last month she spent 120% of her budget, and the bill was \$60. What is Jada's monthly budget? Explain or show your reasoning.

(from Unit 3, Lesson 11)

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5. Which is a better deal, 5 tickets for \$12.50 or 8 tickets for \$20.16? Explain your reasoning.

(from Unit 3, Lesson 9)

6. An athlete runs 8 miles in 50 minutes on a treadmill. At this rate:

- a. How long will it take the athlete to run 9 miles?
- b. How far can the athlete run in 1 hour?

(from Unit 3, Lesson 8)

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Unit 3, Lesson 13: Benchmark Percentages

Let's contrast percentages and fractions.

13.1: What Percentage Is Shaded?



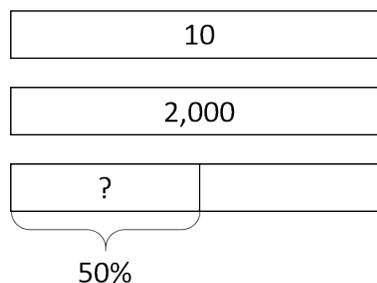
What percentage of each diagram is shaded?

13.2: Liters, Meters, and Hours

1.
 - a. How much is 50% of 10 liters of milk?
 - b. How far is 50% of a 2,000-kilometer trip?
 - c. How long is 50% of a 24-hour day?
 - d. How can you find 50% of any number?

2.
 - a. How far is 10% of a 2,000-kilometer trip?
 - b. How much is 10% of 10 liters of milk?
 - c. How long is 10% of a 24-hour day?
 - d. How can you find 10% of any number?

3.
 - a. How long is 75% of a 24-hour day?
 - b. How far is 75% of a 2,000-kilometer trip?
 - c. How much is 75% of 10 liters of milk?
 - d. How can you find 75% of any number?



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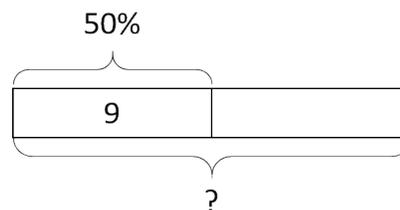
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13.3: Nine is . . .

Explain how you can calculate each value mentally.

1. 9 is 50% of what number?
2. 9 is 25% of what number?
3. 9 is 10% of what number?
4. 9 is 75% of what number?
5. 9 is 150% of what number?



13.4: Matching the Percentage

Match the percentage that describes the relationship between each pair of numbers. One percentage will be left over. Be prepared to explain your reasoning.

- | | |
|--------------------------------|--------|
| 1. 7 is what percentage of 14? | • 4% |
| 2. 5 is what percentage of 20? | • 10% |
| 3. 3 is what percentage of 30? | • 25% |
| 4. 6 is what percentage of 8? | • 50% |
| 5. 20 is what percentage of 5? | • 75% |
| | • 400% |

Are you ready for more?

- What percentage of the world's current population is under the age of 14?
- How many people is that?
- How many people are 14 or older?

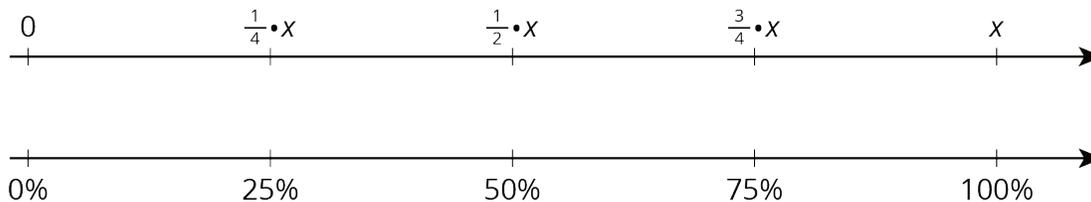
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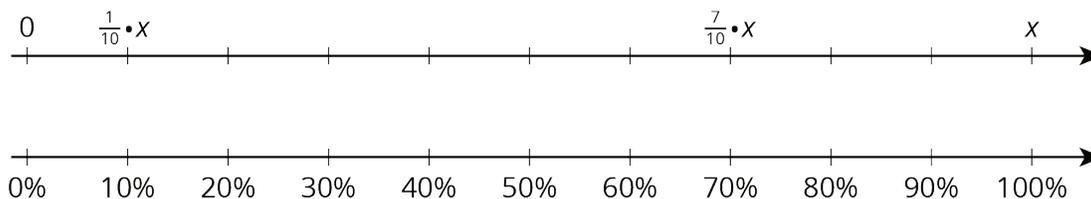
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Lesson 13 Summary

Certain percentages are easy to think about in terms of fractions.



- 25% of a number is always $\frac{1}{4}$ of that number.
For example, 25% of 40 liters is $\frac{1}{4} \cdot 40$ or 10 liters.
- 50% of a number is always $\frac{1}{2}$ of that number.
For example, 50% of 82 kilometers $\frac{1}{2} \cdot 82$ or 41 kilometers.
- 75% of a number is always $\frac{3}{4}$ of that number.
For example, 75% of 1 pound is $\frac{3}{4}$ pound.
- 10% of a number is always $\frac{1}{10}$ of that number.
For example, 10% of 95 meters is 9.5 meters.
- We can also find multiples of 10% using tenths.
For example, 70% of a number is always $\frac{7}{10}$ of that number, so 70% of 30 days is $\frac{7}{10} \cdot 30$ or 21 days.



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Unit 3, Lesson 13: Benchmark Percentages

- How can you find 50% of a number quickly in your head?
 - Andre lives 1.6 km from school. What is 50% of 1.6 km?
 - Diego lives $\frac{1}{2}$ mile from school. What is 50% of $\frac{1}{2}$ mile?
- There is a 10% off sale on laptop computers. If someone saves \$35 on a laptop, what was its original cost? If you get stuck, consider using the table.

savings (dollars)	percentage
35	10
?	100

- Explain how to calculate these mentally.
 - 15 is what percentage of 30?
 - 3 is what percentage of 12?
 - 6 is what percentage of 10?
- Noah says that to find 20% of a number he divides the number by 5. For example, 20% of 60 is 12, because $60 \div 5 = 12$. Does Noah's method always work? Explain why or why not.
- Diego has 75% of \$10. Noah has 25% of \$30. Diego thinks he has more money than Noah, but Noah thinks they have an equal amount of money. Who is right? Explain your reasoning.

(from Unit 3, Lesson 10)

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6. Lin and Andre start walking toward each other at the same time from opposite ends of 22-mile walking trail. Lin walks at a speed of 2.5 miles per hour. Andre walks at a speed of 3 miles per hour.

Here is a table showing the distances traveled and how far apart Lin and Andre were over time. Use the table to find how much time passes before they meet.

elapsed time (hour)	Lin's distance (miles)	Andre's distance (miles)	distance apart (miles)
0	0	0	22
1	2.5	3	16.5
			0

(from Unit 3, Lesson 8)

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Unit 3, Lesson 14: Solving Percentage Problems

Let's solve more percentage problems.

14.1: Number Talk: Multiplication with Decimals

Find the products mentally.

$$6 \cdot (0.8) \cdot 2$$

$$(4.5) \cdot (0.6) \cdot 4$$

14.2: Coupons

Han and Clare go shopping, and they each have a coupon. Answer each question and show your reasoning.

1. Han buys an item with a normal price of \$15, and uses a 10% off coupon. How much does he save by using the coupon?



2. Clare buys an item with a normal price of \$24, but saves \$6 by using a coupon. For what percentage off is this coupon?

Are you ready for more?

Clare paid full price for an item. Han bought the same item for 80% of the full price. Clare said, "I can't believe I paid 125% of what you paid, Han!" Is what she said true? Explain.

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14.3: Info Gap: Music Devices

Your teacher will give you either a problem card or a data card. Do not show or read your card to your partner.

If your teacher gives you the *problem card*:

1. Read your card silently and think about what you need to know to be able to answer the questions.
2. Ask your partner for the specific information that you need.
3. Explain how you are using the information to solve the problem.
4. Solve the problem and show your reasoning to your partner.

If your teacher gives you the *data card*:

1. Read your card silently.
2. Ask your partner "*What specific information do you need?*" and wait for them to *ask* for information.

If your partner asks you for information that is not on the card, do not do the calculations for them. Tell them you don't have that information.

3. Have them explain "*Why do you need that information?*" before telling them the information.
4. After your partner solves the problem, ask them to explain their reasoning, even if you understand what they have done.

Both partners should record a solution to the problem.

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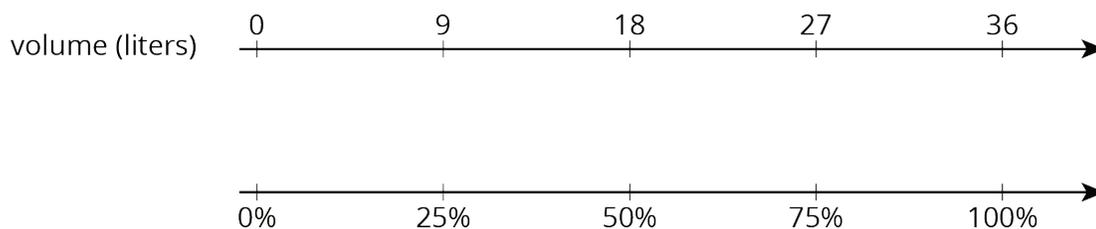
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Lesson 14 Summary

A pot can hold 36 liters of water. What percentage of the pot is filled when it contains 9 liters of water?

Here are two different ways to solve this problem:

- Using a double number line:



We can divide the distance between 0 and 36 into four equal intervals, so 9 is $\frac{1}{4}$ of 36, or 9 is 25% of 36.

- Using a table:

volume (liters)	percentage
36	100
9	25

Two curved arrows labeled $\cdot \frac{1}{4}$ point from the 36 and 100 row down to the 9 and 25 row, indicating multiplication by $\frac{1}{4}$.

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Unit 3, Lesson 14: Solving Percentage Problems

- For each problem, explain or show your reasoning.
 - 160 is what percentage of 40?
 - 40 is 160% of what number?
 - What number is 40% of 160?
- A store is having a 20%-off sale on all merchandise. If Mai buys one item and saves \$13, what was the original price of her purchase? Explain or show your reasoning.
- The original price of a scarf was \$16. During a store-closing sale, a shopper saved \$12 on the scarf. What percentage discount did she receive? Explain or show your reasoning.
- Select **all** the expressions whose value is larger than 100.
 - 120% of 100
 - 50% of 150
 - 150% of 50
 - 20% of 800
 - 200% of 30
 - 500% of 400
 - 1% of 1,000
- An ant travels at a constant rate of 30 cm every 2 minutes.
 - At what pace does the ant travel per centimeter?
 - At what speed does the ant travel per minute?

(from Unit 3, Lesson 8)

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6. Is $3\frac{1}{2}$ cups more or less than 1 liter? Explain or show your reasoning. (Note: 1 cup \approx 236.6 milliliters)

(from Unit 3, Lesson 4)

7. Name a unit of measurement that is about the same size as each object.

a. The distance of a doorknob from the floor is about 1 _____.

b. The thickness of a fingernail is about 1 _____.

c. The volume of a drop of honey is about 1 _____.

d. The weight or mass of a pineapple is about 1 _____.

e. The thickness of a picture book is about 1 _____.

f. The weight or mass of a buffalo is about 1 _____.

g. The volume of a flower vase is about 1 _____.

h. The weight or mass of 20 staples is about 1 _____.

i. The volume of a melon is about 1 _____.

j. The length of a piece of printer paper is about 1 _____.

(from Unit 3, Lesson 2)

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Unit 3, Lesson 15: Finding This Percent of That

Let's solve percentage problems like a pro.

15.1: Number Talk: Decimals

Find the value of each expression mentally.

$$(0.23) \cdot 100$$

$$50 \div 100$$

$$145 \cdot \frac{1}{100}$$

$$7 \div 100$$

15.2: Audience Size

A school held several evening activities last month—a music concert, a basketball game, a drama play, and literacy night. The music concert was attended by 250 people. How many people came to each of the other activities?

1. Attendance at a basketball game was 30% of attendance at the concert.
2. Attendance at the drama play was 140% of attendance at the concert.
3. Attendance at literacy night was 44% of attendance at the concert.

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Are you ready for more?

50% of the people who attended the drama play also attended the music concert. What percentage of the people who attended the music concert also attended the drama play?

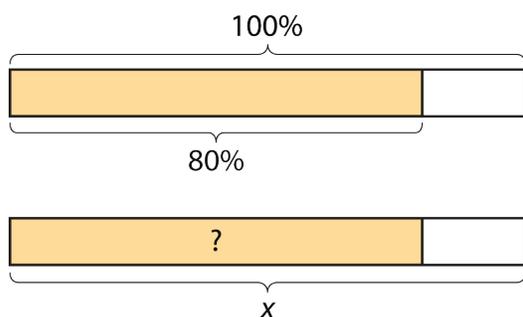
15.3: Everything is On Sale

During a sale, every item in a store is 80% of its regular price.

1. If the regular price of a T-shirt is \$10, what is its sale price?
2. The regular prices of five items are shown here. Find the sale price of each item.

	item 1	item 2	item 3	item 4	item 5
regular price	\$1	\$4	\$10	\$55	\$120
sale price					

3. You found 80% of many values. Was there a process you repeated over and over to find the sale prices? If so, describe it.



4. Which of the following expressions could be used to find 80% of x ? Be prepared to explain your reasoning.

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$$\frac{8}{100} \cdot x$$

$$\frac{8}{10} \cdot x$$

$$\frac{8}{5} \cdot x$$

$$80 \cdot x$$

$$(0.8) \cdot x$$

$$\frac{80}{100} \cdot x$$

$$\frac{4}{10} \cdot x$$

$$\frac{4}{5} \cdot x$$

$$8 \cdot x$$

$$(0.08) \cdot x$$

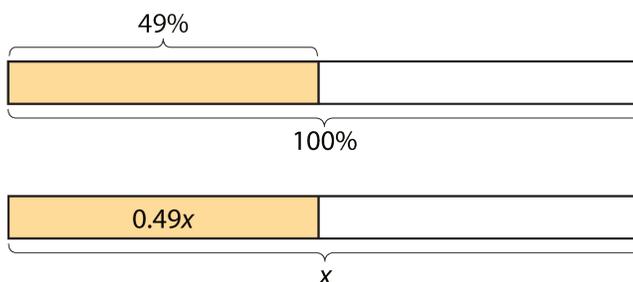
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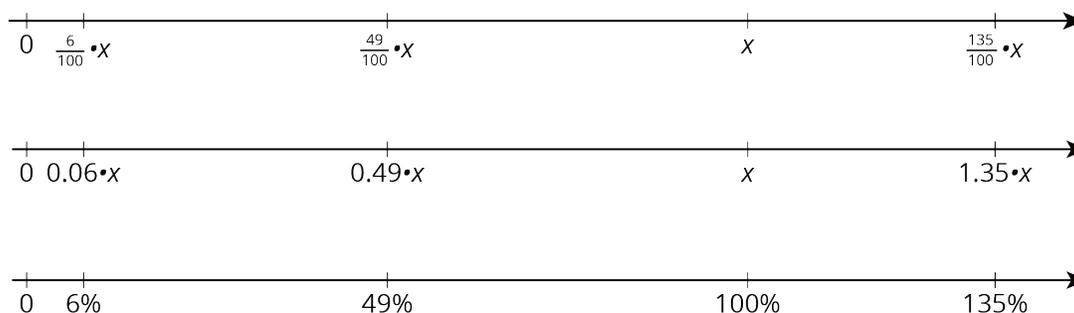
Lesson 15 Summary

To find 49% of a number, we can multiply the number by $\frac{49}{100}$ or 0.49.



To find 135% of a number, we can multiply the number by $\frac{135}{100}$ or 1.35.

To find 6% of a number, we can multiply the number by $\frac{6}{100}$ or 0.06.



In general, to find $P\%$ of x , we can multiply:

$$\frac{P}{100} \cdot x$$

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- 12% of 700

5. Complete each statement.

a. 20% of 60 is _____

d. 50% of 90 is _____

b. 25% of _____ is 6

e. 10% of _____ is 7

c. _____% of 100 is 14

f. 30% of 70 is _____

(from Unit 3, Lesson 14)

6. A shopper needs 24 sandwich rolls. The store sells identical rolls in 2 differently sized packages. They sell a six-pack for \$5.28 and a four-pack for \$3.40. Should the shopper buy 4 six-packs or 6 four-packs? Explain your reasoning.

(from Unit 3, Lesson 9)

7. On a field trip, there are 3 chaperones for every 20 students. There are 92 people on the trip. Answer these questions. If you get stuck, consider using a tape diagram.

a. How many chaperones are there?

b. How many children are there?

(from Unit 2, Lesson 15)

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Unit 3, Lesson 16: Finding the Percentage

Let's find percentages in general.

16.1: True or False: Percentages

Is each statement true or false? Be prepared to explain your reasoning.

- 25% of 512 is equal to $\frac{1}{4} \cdot 500$.
- 90% of 133 is equal to $(0.9) \cdot 133$.
- 30% of 44 is equal to 3% of 440.
- The percentage 21 is of 28 is equal to the percentage 30 is of 40.

16.2: Jumping Rope

A school held a jump-roping contest. Diego jumped rope for 20 minutes.

- Jada jumped rope for 15 minutes. What percentage of Diego's time is that?
- Lin jumped rope for 24 minutes. What percentage of Diego's time is that?
- Noah jumped rope for 9 minutes. What percentage of Diego's time is that?
- Record your answers in this table. Write the quotients in the last column as decimals.

	time (minutes)	percentage	time \div 20
Diego	20	100	$\frac{20}{20} = 1$
Jada	15		$\frac{15}{20} =$
Lin	24		$\frac{24}{20} =$
Noah	9		$\frac{9}{20} =$

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5. What do you notice about the numbers in the last two columns of the table?

16.3: Restaurant Capacity

A restaurant has a sign by the front door that says, "Maximum occupancy: 75 people." Answer each question and explain or show your reasoning.

1. What percentage of its capacity is 9 people?

2. What percentage of its capacity is 51 people?

3. What percentage of its capacity is 84 people?

Are you ready for more?

Water makes up about 71% of the Earth's surface, while the other 29% consists of continents and islands. 96% of all the Earth's water is contained within the oceans as salt water, while the remaining 4% is fresh water located in lakes, rivers, glaciers, and the polar ice caps.

If the total volume of water on Earth is 1,386 million cubic kilometers, what is the volume of salt water? What is the volume of fresh water?

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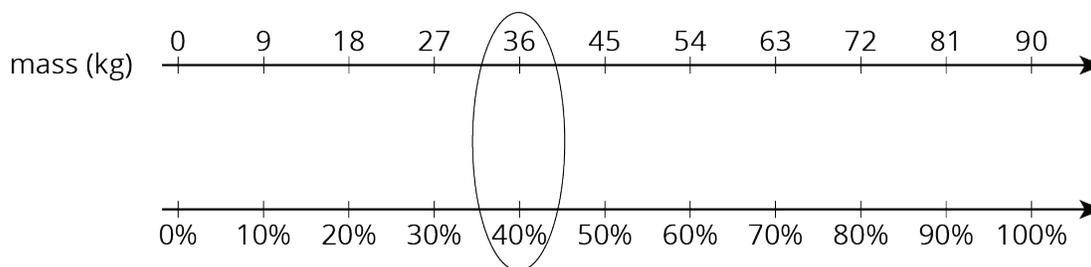
Lesson 16 Summary

What percentage of 90 kg is 36 kg? One way to solve this problem is to first find what percentage 1 kg is of 90, and then multiply by 36.

mass (kg)	percentage
90	100
1	$\frac{1}{90} \cdot 100$
36	$\frac{36}{90} \cdot 100$

Diagram illustrating the calculation of percentages for different masses. On the left, green arrows point from the mass values (90, 1, 36) to the corresponding percentage values, with labels $\cdot \frac{1}{90}$ and $\cdot 36$. On the right, green arrows point from the percentage values back to the mass values, with labels $\cdot \frac{1}{90}$ and $\cdot 36$.

From the table we can see that 1 kg is $(\frac{1}{90} \cdot 100)$ %, so 36 kg is $(\frac{36}{90} \cdot 100)$ % or 40% of 90. We can confirm this on a double number line:



In general, to find what percentage a number C is of another number B is to calculate $\frac{C}{B}$ of 100%. We can find do that by multiplying:

$$\frac{C}{B} \cdot 100$$

Suppose a school club has raised \$88 for a project but needs a total of \$160. What percentage of its goal has the club raised?

To find what percentage \$88 is of \$160, we find $\frac{88}{160}$ of 100% or $\frac{88}{160} \cdot 100$, which equals $\frac{11}{20} \cdot 100$ or 55. The club has raised 55% of its goal.

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Unit 3, Lesson 16: Finding the Percentage

1. A sign in front of a roller coaster says "You must be 40 inches tall to ride." What percentage of this height is:
 - a. 34 inches?
 - b. 54 inches?
2. At a hardware store, a tool set normally costs \$80. During a sale this week, the tool set costs \$12 less than usual. What percentage of the usual price is the savings? Explain or show your reasoning.
3. A bathtub can hold 80 gallons of water. The faucet flows at a rate of 4 gallons per minute. What percentage of the tub will be filled after 6 minutes?
4. The sale price of every item in a store is 85% of its usual price.
 - a. The usual price of a backpack is \$30, what is its sale price?
 - b. The usual price of a sweatshirt is \$18, what is its sale price?

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c. The usual price of a soccer ball is \$24.80, what is its sale price?

(from Unit 3, Lesson 15)

5. A shopper needs 48 hot dogs. The store sells identical hot dogs in 2 differently sized packages. They sell a six-pack of hot dogs for \$2.10, and an eight-pack of hot dogs for \$3.12. Should the shopper buy 8 six-packs, or 6 eight-packs? Explain your reasoning.

(from Unit 3, Lesson 9)

6. Elena is 56 inches tall.

a. What is her height in centimeters? (Note: 100 inches = 254 centimeters)

b. What is her height in meters?

(from Unit 3, Lesson 4)

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Unit 3, Lesson 17: Painting a Room

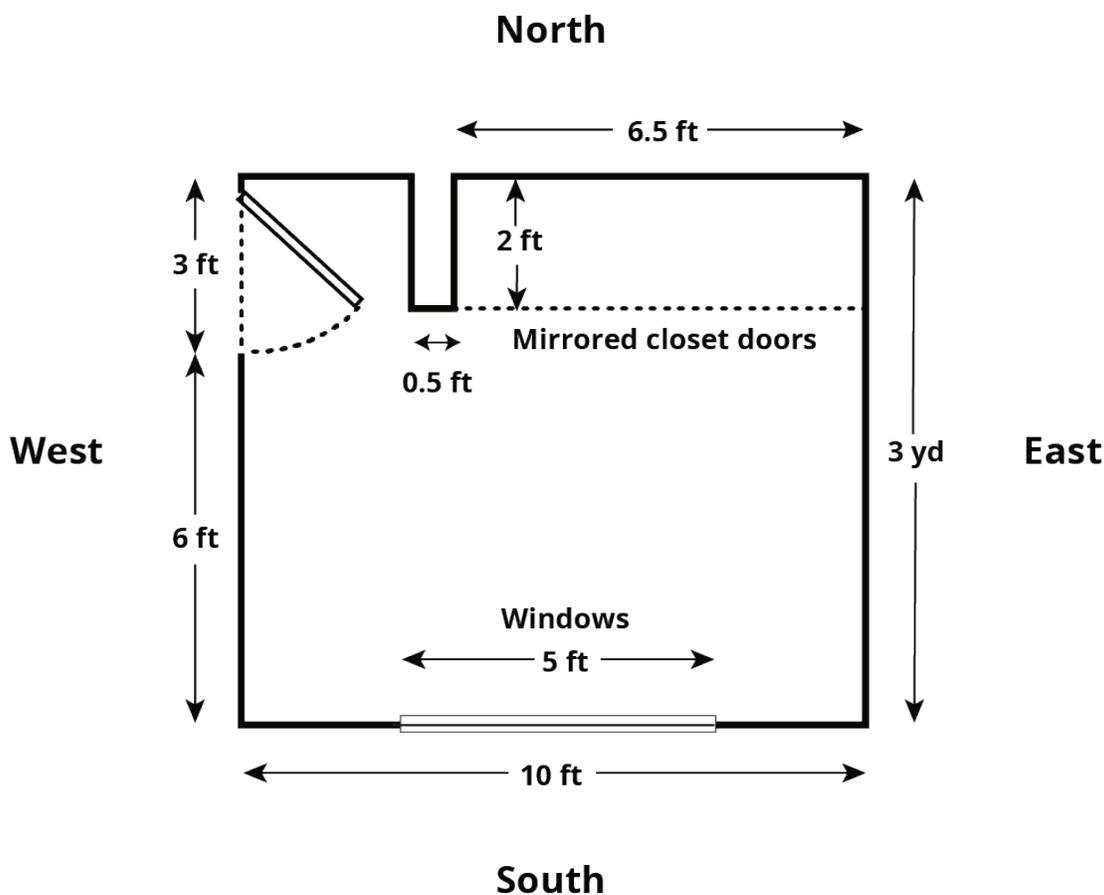
Let's see what it takes to paint a room.

17.1: Getting Ready to Paint

What are some tools that are helpful when painting a room?

17.2: How Much It Costs to Paint

Here is the floor plan for a bedroom:



NAME

DATE

PERIOD

Imagine you are planning to repaint all the walls in this room, including inside the closet.

- The east wall is 3 yards long.
- The south wall is 10 feet long but has a window, 5 feet by 3 feet, that does not need to be painted.
- The west wall is 3 yards long but has a door, 7 feet tall and 3 feet wide, that does not need to be painted.
- The north wall includes a closet, 6.5 feet wide, with floor-to-ceiling mirrored doors that do not need to be painted. There is, however, a smaller wall between the west wall and the closet that does need to be painted on all sides. The wall is 0.5 feet wide and extends 2 feet into the room.
- The ceiling in this room is 8 feet high.
- All of the corners are right angles.

1. If you paint all the walls in the room, how many square feet do you need to cover?

2. An advertisement about the paint that you want to use reads: "Just 2 quarts covers 175 square feet!" If you need to apply two coats of paint on all the walls, how much paint do you need to buy?

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3. Paint can only be purchased in 1-quart, 1-gallon, and 5-gallon containers. How much will all supplies for the project cost if the cans of paint cost \$10.90 for a quart, \$34.90 for a gallon, and \$165.00 for 5 gallons?

4. You have a coupon for 20% off all quart-sized paint cans. How does that affect the cost of the project?

17.3: How Long It Takes to Paint

After buying the supplies, you start painting the east wall. It takes you 96 minutes to put two coats of paint on that wall (not including a lunch break between the two coats).

1. Your friend stops by to see how you are doing and comments that you are 25% finished with the painting. Are they correct?

2. Your friend offers to help you with the rest of the painting. It takes the two of you 150 more minutes of painting time to finish the entire room. How much time did your friend save you?